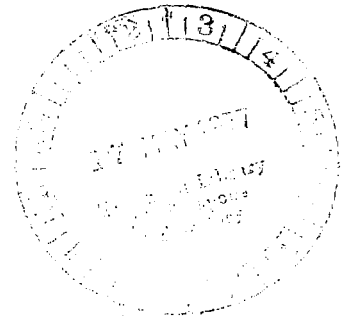


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# **SYSTEMS RELIABILITY ISSUES FOR FUTURE AIRCRAFT**

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**A workshop sponsored by  
Ames Research Center  
held at  
Massachusetts Institute of Technology  
Cambridge, Massachusetts  
August 18-20, 1975**



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## FOREWORD

The trend toward increased automatic control of aircraft through the use of sophisticated on-board digital computers is clearly evidenced by the nationwide efforts on active control technology and integration of flight control and vehicle configuration. Future high performance aircraft, both civilian and military, will be characterized by an increase in the number of sensors, aerodynamic control surfaces, and direct thrust devices used. Improved performance achieved through a reduction in weight is being sought for large transport aircraft by having active attitude and mode controls allow reduced margins of open-loop rigid and flexible stability. Great maneuverability with adequate ride and handling qualities is being sought for STOL and VTOL vehicles through various methods of providing high lift, which usually increase the number of actuators to be controlled.

The increase in functions being sought for aircraft increases the number of sensors and actuators on board. The multiplicity of hardware alone induces a need for automatic processing — of data or of control — to aid the pilot. The multiplicity of choice that naturally arises also leads to increased computation and automation. Fortunately, modern control systems theory appears to be well suited for the design of such complex systems. When the criticality of survival of some of these functions is considered, however, the picture changes: there is a great increase in hardware, some increase in choice, but not yet a corresponding growth in constructive and relevant theory. Nevertheless, the new organizational and functional questions are control and systems theoretic in nature and should be addressed by the control system community.

A workshop bringing about 30 government, university, and industry researchers to encourage the necessary dialog was organized by MIT's Electronic Systems Laboratory, sponsored by Ames Research Center, NASA, and held at MIT August 18-20, 1975. It consisted of formal and informal presentations of surveys and theoretical and practical developments and of discussions. This report contains the formal proceedings of that workshop.

The first group of papers relates to implementing advanced systems. After Doolin's introduction pointing out the opportunities for control-system theoretic contributions to future flight control, Taylor reviews NASA's active control program. The review points out the performance benefits of active control as well as the stringent reliability requirements. Next Meyer describes a series of experiments planned at Ames Research Center for V/STOL (vertical or short takeoff and landing) aircraft. These experiments are to determine the efficacy of a new versatile digital-computer-oriented flight-control system for the various active control modes associated with V/STOL flight. Another approach to solving the problem of a flexible digital flight-control system is given next by Stein (which was delivered by Yore). It, too, aims at satisfying the requirements of operating throughout a flight envelope with a structure that can allow extra provisions for reliable operation. In a similar vein, Astrom discusses some of the research carried out at the Lund Institute in Sweden on self-tuning regulators and their experimental use in

controlling sea ships. Montgomery's paper, the final one of this group, addresses the problem of failure detection and the computational experiments being conducted at Langley Research Center for feasible implementation of maximum likelihood methods.

Two informal presentations are not included in these proceedings. Berger discussed (see ref. 1) the present approach to reliable operation as employed by McDonnell-Douglas. Requirements are satisfied by quadruple redundancy of hardware. Particular advantage is taken of built-in test equipment. In some cases, this "firmware" detects degradation in hardware performance and alerts the flight crew or maintenance accordingly. In a second informal discussion, Clark (ref. 2) recounted experiences of using observers to detect incipient failures in inertial instruments.

Saridis' paper on learning systems provides a transition between the practically oriented papers above and the theoretical discussions to come. This paper, together with that of Tse and an informal discussion by Ho of team decision theory, provided an excellent view of the status of theory and heuristics in combined estimation and control. Learning is used to develop an input-output map when the intervening system is assumed to be inexplicably complex. Dual control (discussed next) deserves consideration when the mission is sensitive to precise knowledge of parameters and the parameters in turn can best be determined if the system is perturbed. Team problems model decentralized control and decentralized information where different controllers share the same goal. The formidable difficulties in dual control and team problems are illustrated by the paucity of definite quantitative results to date.

The extensions to gain margin theorems in Athans' paper derive from considerations of decentralized control and robustness research currently under investigation in the Electronic Systems Laboratory. Kleinman's paper reviews the successes and promise of modern control systems theory in modeling human behavior. The text for the presentation is not available, but the figures that are reproduced herein substantially convey the information. The final paper along control systems theory lines is Wonham's review of the results in linear multivariable theory obtained both by his geometric approach and by the transfer matrix approaches of Wolovich and Rosenbrock. These proceedings conclude with two excellent reviews of current research efforts in nonlinear filtering and failure detection by Rhodes and Willsky.

The discussion periods emphasized how very complicated software becomes as soon as an operation is automated. Lacking the great tolerance to error that a human operator provides means that the conditions of operation must be specified in extreme detail. It was recognized that the required automation would be unthinkable except for digital computation.

Although hardware people did not attend the workshop, their claim to better reliability than software can give was voiced. Clearly, software-generated functional reliability will not replace all hardware replication, yet software and computation reliability is being improved. The confidence in the reliability of hardware operation is largely due to the increased use

of relatively simple software or "firmware" as in builtin test equipment. Hence the argument of hardware vs. software is really one of degree. It is an argument that cannot be resolved definitively as yet because we do not now have appropriate principles by which to organize the logical structure of the software.

As principles of organization for the synthesis of functional reliability are missing, so are criteria and methods for its analysis. Available failure modes assessment and event tree methods, of course, provide much information. They do not give rise to a dynamics that would enable a convenient process for optimization or tradeoff studies. Berger recalled that Honeywell had once begun a study that could have led to a dynamic finite-state space method of analysis (ref. 3). Apparently the work was discontinued.

Participants on the aeronautics side had the benefit of an excellent review of the status of most control system theory. Participants on the control systems side got an excellent perspective on expectations and open problems in aeronautics. People of all backgrounds represented joined in the lively discussions. If this dialog concerning principles and methods of designing for reliability persists, it will be to the great benefit both of control-system theory and of aeronautics, and the workshop will have served its purpose.

This workshop was made possible through NASA grant NSG-2076. We are grateful to Mr. R. A. Osborne, Ms. B. Peacock, and Mr. W. H. Lee for their tireless efforts in organizing the workshop and for keeping everybody happy.

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# TABLE OF CONTENTS

	<u>Page</u>
FOREWORD . . . . .	iii
Michael Athans and Brian F. Doolin	
Massachusetts Institute of Technology and Ames Research Center	
INTRODUCTION . . . . .	1
Brian F. Doolin, Ames Research Center, NASA	
APPLICATION OF ACTIVE CONTROLS TO CIVIL TRANSPORT AIRCRAFT . . . . .	9
Lawrence W. Taylor, Jr., Langley Research Center, NASA	
NASA AMES ACTIVE CONTROL AIRCRAFT FLIGHT EXPERIMENTS (ACA) PROGRAM . . .	21
George Meyer and William R. Wehrend, Ames Research Center, NASA	
ADAPTIVE CONTROL LAWS FOR F-8 FLIGHT TEST . . . . .	27
G. Stein, G. L. Hartmann, and R. C. Hendrick	
Honeywell, Inc., Minneapolis, Minnesota	
SELF-TUNING REGULATORS . . . . .	51
K. J. Åström, Lund Institute of Technology, Sweden	
FAILURE DETECTION AND CONTROL-SYSTEM RECONFIGURATION: PAST, PRESENT,	
AND FUTURE . . . . .	69
Raymond C. Montgomery, Langley Research Center, NASA	
FACTS AND FICTION OF LEARNING SYSTEMS . . . . .	79
George N. Saridis, Purdue University, Ind.	
STATUS OF DUAL CONTROL THEORY . . . . .	105
Edison Tse, Stanford University, Calif.	
CLOSED-LOOP STRUCTURAL STABILITY FOR LINEAR-QUADRATIC OPTIMAL SYSTEMS. .	123
Poh Kam Wong and Michael Athans	
Massachusetts Institute of Technology, Mass.	
ANALYSIS OF PILOT-AIRCRAFT PERFORMANCE AND RELIABILITY VIA THE	
APPLICATION OF MAN-MACHINE MODELS . . . . .	141
David L. Kleinman, University of Connecticut, Con.	
RECENT PROGRESS IN MULTIVARIABLE CONTROL . . . . .	157
W. M. Wonham, University of Toronto, Canada	
A SUMMARY OF NONLINEAR FILTERING . . . . .	165
Ian B. Rhodes, Washington University, Mo.	
A SURVEY OF DESIGN METHODS FOR FAILURE DETECTION IN DYNAMIC SYSTEMS . .	181
Alan S. Willsky, Massachusetts Institute of Technology, Mass.	
LIST OF PARTICIPANTS . . . . .	209



## INTRODUCTION

Brian F. Doolin

Ames Research Center, NASA

My thanks to all of you for coming to participate in what promises to be a stimulating and productive workshop. My particular thanks to our chairman, Mike Athans, for his recommendations that spawned this workshop and for his labors that brought it about.

My purpose this morning is simply introductory and motivational. The workshop's title, "Systems Reliability Issues for Future Aircraft," covers a call for help in the task of using systems theory to improve the reliability of aircraft flight. What is sought is direction in developing a unified analytic approach to exploit the couplings between the many sources of information and between the many sources of control in airplanes.

Much of the attention in Control Systems Engineering has focused on developing what is by now an extensive and effective theory to analyze and synthesize systems that are relatively simple structurally. The success of that theory, together with the improvements evident in the technology of digital computation, emboldens us to consider directly the challenge thrust by aeronautics. The source of the challenge is the increase in the number of control functions being demanded of many types of aircraft.

Control Systems Engineering must find a rational approach to structuring these control functions so as to secure both order in their increased complexity and flexibility in their use. The task is important in that its success will allow other aircraft technologies to achieve their goals of performance and economy by reducing the replication of components that reliability considerations will otherwise require.

For motivation, I will point out opportunities for control that some aircraft provide, and lean on a little history to show the interplay between human and automatic operation. Many of you are intimately familiar with these topics and will discuss some of them to much greater depth. I ask your forbearance.

Figures 1 to 4 are of aircraft or aircraft models that have been flown or tested at Ames. They illustrate various opportunities for aerodynamic or propulsive control. Figures 1 and 2 show some aerodynamic controls. Figure 1 shows our C-141 aircraft with its flaps, at the trailing edge of the

wings, partially deployed — the flap is split into two sections. The ailerons that make the aircraft roll are located farther along the trailing edge of the wing. Besides their primary job of rolling the aircraft or increasing its lift at the lower flight speeds, these surfaces can be used to shape the airloads on the wing. Leading-edge surfaces are also used. For various reasons, the vertical control surface of the tail is sometimes split.

Some aircraft also have small surfaces — canard surfaces — on the forward part of the fuselage. An example is the wind-tunnel model in figure 2. They are used to help control body bending or even to improve aircraft control at low speeds. Similar controls have been installed on special aircraft to generate controllable side forces.

Perhaps figure 3 portrays the propulsion man's dream of what aircraft should look like: a lift-fan V/STOL vehicle, models of which have undergone wind-tunnel testing. Each of six jet engines drives a turbine that sucks in air. The two rear engines have shrouds that deflect the air downward during flight at low speeds and hover. They are the only engines operating in normal flight. Louvers below the fans modulate the flow for attitude control at speeds too low for normal aerodynamic control. You can well imagine that piloted digital simulation models have been used to investigate various engine failures and the ability of pilots to handle the failures.

Figure 4 shows the Augmentor Wing Research Aircraft. These flaps are not ordinary aerodynamic surfaces, but ducts containing a flow of cool air from the engines. This flow also entraps air flowing over the wing. Perhaps you can see this nozzle on the engine. There are four of them in all, two on each engine. They can be rotated to deflect the direct hot air flow of the engines from straight back to straight down and even somewhat forward. The combination of direct cold engine air, entrapped flow over the wing, and directed thrust gives enough lift for the aircraft to descend steeply but gently at 80 or 90 mph. These photographs are shown to illustrate that an increasing number of aerodynamic and propulsive controls are evident on present and future aircraft.

If the pilot were responsible for using all these controls, he would be faced with enormous tasks of actions and decisions. Such regards have been the basic cause for considering automation on aircraft. Figure 5 summarizes the basis qualitatively: If things happen too fast for the pilot to handle, or if there are just too many things to keep track of and make judgments on, then it is necessary to give him relief. On the other hand, if he only has minor functions to fiddle with over an extended period of time, he gets bored and calls for help.

Figure 6 illustrates the time-scale spectrum of operations associated with flight. The middle column refers to the aircraft, and that on the right to the pilot. The durations of flight operations are from the order of hours for the cruise portion of flight through mission phases like takeoff and approach and landing that take minutes. Duration of flight regimes or mission phases overlaps the duration of flight-path dynamics whose time constants are minutes to seconds. Then attitude dynamics takes over with short



periods of 1 to 2 sec. Structural effects overlap here with limit cycle flutter periods reaching down to fractions of a second. The spectrum gives an understandable ordering of operation: each shorter period motion (except for the structural noise) acts as the driving function for the next slower motion.

The human response is characterized in the next column, which shows that the pilot's performance deteriorates on both sides of periods of the order of minutes. What could not be shown on the figure is a plot of human activity vs. operational phase — how many things are to be done or decisions to be made. Those functions depend on failure conditions (whose response times fall in the lower part of the time scale) and on the design of the control and information systems, including help given the pilot by automatic systems.

The pilot wants control where he is most effective so long as the load is reasonably organized and not excessive. He not only wants control, but the total package is most effective with him operating in his best region, as the little history of the early days of flight (fig. 7) illustrates (ref. 1).

The first time period in the figure corresponds to the earliest days of this century when the vehicle was being developed as a tool for transportation. Flights were short and limited to the best weather. As flight times increased, navigation followed roads and available maps. Since you cannot go very far or often without running into weather giving poor visibility, the next pilot aids had to do with his receiving information via radio, first voice and then also beacon signals that provided the earliest skyways. With improved vehicles and trip times, the pilot no longer had to pay constant attention to routine and dull tasks since autopilots now could maintain a status quo.

The corresponding history of vehicle stabilization and control is conveyed in the second part of figure 7. A major contribution of the Wright brothers was to relax the inherent stability that earlier designers were building into their vehicles. When the pilot was allowed to assume some of the stabilization duties, path control became reasonable. Stabilization required too much agility and attention — it was exhausting. So Sperry relieved the pilot of some of this task with his gyrostabilizer in 1913. He improved stabilization while retaining the improved configuration technology. This mechanism was done away with once the existence of a tradeoff between stability and control was understood and airframe designers built compromises into the configuration. The compromises were haphazard until the mid-thirties when researchers conceived of handling qualities as a design guide and began to regularize design conditions.

Figure 8 brings us up to date. Our present highly developed airway system that enabled long-distance flight began with the introduction of simple ground beacons. Improvements in onboard instrumentation and computation have recently relaxed the need for strict following of the one-dimensional airway system. Adding one degree of freedom in flight — two-dimensional, or

area, navigation — is becoming commonplace. In fact, at Ames, four-dimensional feasibility has been demonstrated.

For the remaining items listed in the figures, onboard capability and requirements really increase, each requirement calling for improvements and additions in controls and sensing. All the while, built-in means of supplementing reliability become more important until, with relaxed static stability, we are back to the Wright brothers, and reliability must be superimposed and guaranteed. All these requirements and more are being satisfied, as some of you will be telling us over the next day or so.

Now comes the problem of coping with the complexity of control and the amount of intelligence about status and environment. The onboard system must understand what is occurring, anticipate difficulties, and avoid them.

The issues of reliability and redundancy are being addressed from the systems point of view, as the brief list in figure 9 indicates. Broen (ref. 2) has suggested improvements in handling voting smoothly and in not destroying form when lines of information are lost. The advantages of improved use of information made available by modern estimation methods were discussed by Beard (ref. 3). Montgomery and Price (ref. 4) and Willsky and Jones (ref. 5) are making this approach a reality, as well as teaching us how complex the task can be. Martin and Harding (ref. 6) investigated the type of redundant control occurring in the autopilot design for the augmentor wing and found that strategy of control can be partitioned, a result that provides some basis for a formal division of control authority and also allows backup or redundancy.

Certainly, the beginnings of the intelligent use of available redundancy to reduce the need for replication for a given level of reliability are found in these works of Beard, Montgomery, Willsky, Martin, and others. The systems theoretic basis they use is available. For this type of approach to encompass the total flight-control problem and coordinate available resources in the face of increasing control and sensing requirements, additions to the theory are required. The time is right for these additions now that systems theorists are beginning to pay attention to the structure of large-scale systems.

Flight control seems to provide a good model for a large-scale system because it is more amenable to analytic description than most other examples available. We in the aeronautics business propose that flight control be used as a paradigm, of course, since we are interested in developing the proper theory. Our interest is evidenced by a workshop held last year at Utah State University (ref. 7). The main thrust of that workshop was to seek a common description of large-scale systems.

At the workshop, the best criteria advanced for formally treating problems as large were those that have historically driven us to automatic control, namely, the first three reasons shown in figure 5. The many decisions and operations called for in large systems make it plausible that consideration be given to hierarchic structures for their design. The theoretical

papers of that workshop aimed at building a proper system model by first postulating a hierarchic or at least an aggregated structure. Some authors inquired into the information and control patterns that conserved that structure. Others investigated conditions that preserve some overall stability when interconnections in that structure were torn. But no formalized basis for aggregating or for generating hierarchic structures appeared. Finding it may help uncover a formal approach to a flexible organization we are aiming for. I anticipate the deliberations of the next few days, knowing that, directly or indirectly, they will evoke the conceptual framework to design a reliable control system.

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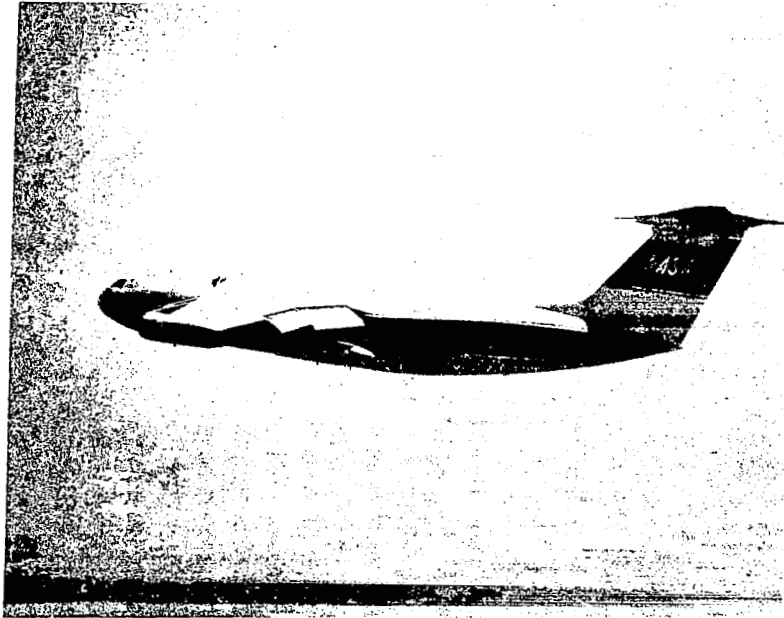


Figure 1.- C-141 Aircraft with flaps partially deployed.

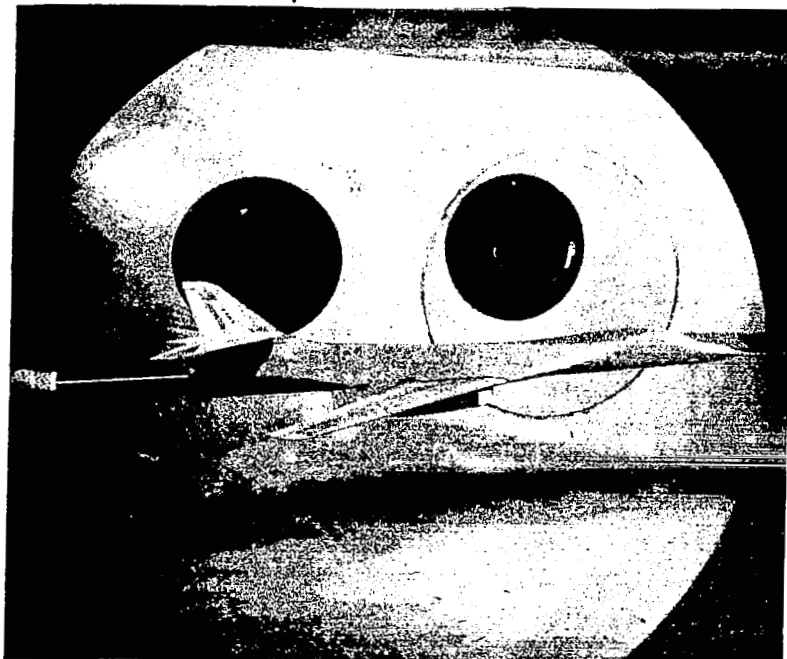


Figure 2.- Wind tunnel model showing canard control surfaces.

## PROPULSION CONCEPT

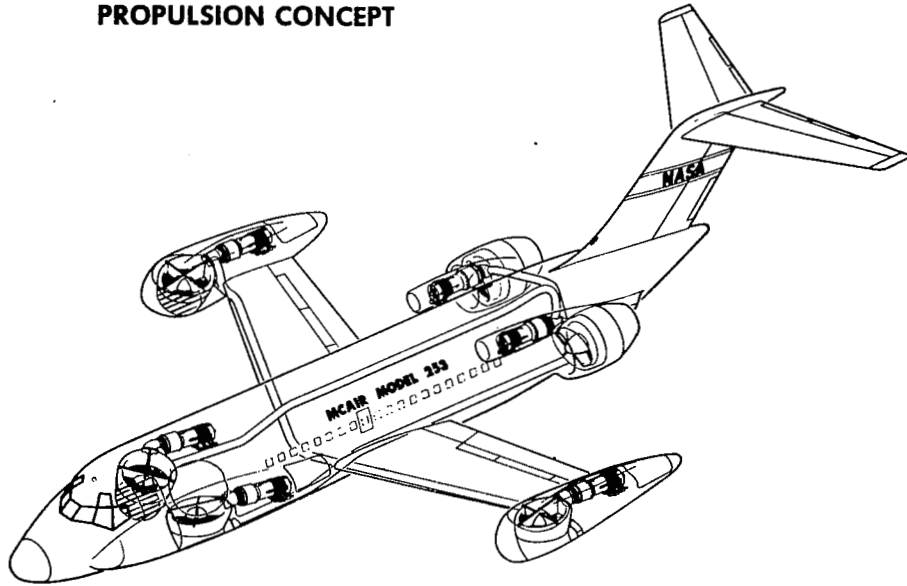


Figure 3.- Schematic drawing of one proposed lift-fan V/STOL aircraft.



Figure 4.- Ames' Augmentor Wing Jet STOL Research Aircraft just before touchdown.

Things happen too fast  
 Too many things to do  
 Too much data to process  
 OR  
 Too little of the above

Figure 5.- Why is automatic control required?

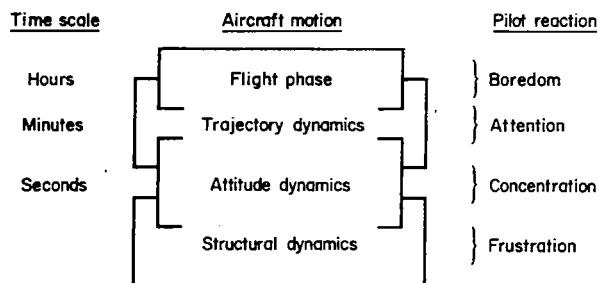


Figure 6.- Time scale of aircraft and pilot operations.

Early vehicle use:

Flight limited by range and visibility  
 Add communication and navigation  
 Relieve pilot by autopilot

Early vehicle development

Let the pilot aid stability  
 Help the pilot by gyrostabilizer  
 Help the pilot by configuration

Figure 7.- Aircraft operations to the '30's.

Flight path requirements:

Route structures  
 Area navigation  
 4-D navigation  
 Weapons delivery  
 Terrain avoidance

Stabilization and control requirements:

Handling qualities improvement  
 Flutter mode control  
 Gust load alleviation  
 Force control  
 Relaxed static stability

Figure 8.- Example growth of control requirements.

Voting preserving structure

Broen

Exploiting information

Beard  
 Montgomery  
 Willsky

Redundant control

Martin

Figure 9.- Failure detection and redundancy utilizing system structure.

# APPLICATION OF ACTIVE CONTROLS TO CIVIL TRANSPORT AIRCRAFT

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## INTRODUCTION

Active controls had its start before the Wright Brothers flew at Kitty Hawk in 1903. In 1894, Hiram Maxim (ref. 1) designed an active control system that used steam-powered gyros connected to steam-powered actuators to stabilize his airplane. The tests he conducted on a constrained rail system which enabled the airplane to go forward but not to lift off met with disaster as the lift created was great enough to break away from the rail system, destroying the aircraft before it could be flown. Most aircraft since have relied on inherent stability, with automatic systems being used to improve handling qualities. A few aircraft, however, have required automatic control systems to enable the pilot to fly the airplanes (i.e., the X-15, B-58, and SR71 airplanes). On today's civil aircraft, certain devices such as yaw dampers and speed stabilizers are required for the safe operation of the aircraft. On the L-1011, for example, the yaw damper must be operable before the airplane can be dispatched for its flight. Over the past several years, the Flight Dynamics Laboratory at Wright-Patterson AFB has promoted the concept of the control configured vehicle (CCV) whereby the control considerations are considered to be an integral part of the preliminary design of an aircraft (ref. 2). It is in this spirit that active controls are being investigated for civil transport aircraft by NASA. NASA has funded a series of design studies (refs. 3-5) in which benefits of active controls were addressed. There remains, however, the task of studying in depth the impact of active controls on civil transport aircraft. It is not the purpose of this paper to give a complete dissertation on the application of active controls but rather to describe some of the very complex problems involved in integrating active controls and to give some notions as to an approach. It is hoped that some of these notions might prove useful to the aircraft designers whose job it is to "put it all together." Figure 1 is a schematic of the job of integrating propulsion, aerodynamics, structures, and controls in a conceptual design. References 6 through 8 describe many of the design considerations involved with active controls not discussed here.

## ACTIVE CONTROLS

Active controls consists of several concepts (figs. 2 and 3), each of which improves some facet of airplane performance. For example, the *reduced static stability system* uses a servoed horizontal stabilizer to provide stability as opposed to a larger fixed horizontal stabilizer. By reducing

the size, weight, and drag of the tail, the airplane can be made lighter and more efficient. It also enables a smaller wing to be used since the horizontal stabilizer can now supplement the wing lift. In a *gust load alleviation system*, flaps and the horizontal stabilizer are actuated to reduce the peak dynamic loads resulting from gusts. The result is that a lighter wing structure can be used, thereby reducing the total weight of the airplane. In much the same way, the *maneuver load alleviation system* reduces wing root bending by causing a lift distribution more concentrated toward the center of the airplane. Again, the reduced structural weight results in improved airplane efficiency. The *flutter suppression system* stabilizes aeroelastic modes that otherwise might flutter by the judicious actuation of control surfaces at strategic locations on the airplane. When applied to a new airplane conceptual design, all of these functions will likely be merged into a single active control system. The individual concepts are useful in synthesizing the control law and in working trade studies during the early design phase.

Another consideration of active controls is the degree to which reliance is placed on the proper functioning of the control system. Active controls can be used in a noncritical function in which case a failure would not be catastrophic or they can be used in a flight critical role so that their successful operation is required to operate the airplane. The reliability, complexity, and cost of control systems having a flight critical role may detract from benefits in efficiency that would otherwise be realized. One can perhaps begin to appreciate the very complex trades yet to be analyzed in detail before one can say with assurance what role active controls should play in future civil transports.

#### NASA ACTIVE CONTROL TECHNOLOGY PROGRAM

The objective of the NASA Active Control Technology Program (fig. 4) is to identify, develop, and validate the controls technology required to integrate active controls in future subsonic civil aircraft. The program (fig. 5) consists of four parts: (1) baseline data that establish data and certain techniques required to apply active controls, (2) validation flight and wind-tunnel tests to assess the accuracy of certain theoretical results, (3) an active controls analysis package that incorporates the necessary analytical techniques into a versatile set of computer programs, and (4) integrated conceptual design studies involving active control. The active control technology deemed to be ready by a time such as 1985 will then be directed toward suitable flight demonstrations in preparation for the application of flight critical active controls on future civil transports.

This paper discusses the approach being taken by NASA as part of the Active Control Technology Program to integrate active controls in the conceptual design phase. The approach is analytical and is depicted by

- A mathematically expressed design criterion such as return on total investment



- Mathematically expressed relationships and constraints involved in the conceptual design process
- A numerical technique that optimizes the design variables with respect to the criterion
- Manual interface to assess discrete alternatives involved in the design

## DESIGN CRITERION

The design criterion should be the same for all aspects of the airplane and should reflect the basis on which the aircraft will be judged in terms of its performance. It must be expressible in mathematical terms and must be a scalar. The design criterion should merge (fig. 6) all of the various concerns of the design, whether these be performance, cost, revenue, safety, noise, pollution, or fuel consumption. Criteria used in the past include direct operating cost, sometimes modified to reflect revenue and depreciation, and return on an investment. Return on investment is of particular interest to the author since ROI is believed to be close to that quantity in which the merit of a civil transport airplane is judged by its user, the airline. For our purposes, the design problem is considered in terms of determining the values of the numerous design variables that minimize the design criterion — return on investment. Many equality and inequality constraints must also be satisfied.

A very useful concept concerning the design criterion is what might be called the "linearized design criterion" (fig. 7). If the design criterion is thought of as a function of the many design variables of the airplane, it is possible to differentiate the design criterion (in concept at least) with respect to these design variables. If it were possible to do this, one would obtain a linear version of the design criterion. A single example is

$$\Delta \text{ROI} = \text{initial cost}/(\$10 \text{ M}) + \text{operating cost}/(\$126/\text{hr}) + \text{weight}/(5657 \text{ lb})$$

Each unit that  $\Delta \text{ROI}$  is increased approximately corresponds to an incremental change in ROI of about 1 percent of return, for the current class of wide-body jets. This linearized design criterion can then be used independently by various design specialists to aid in the assessment of certain alternatives during the design process. In this way, numerous design studies can be performed using the same overall criterion. As a particular conceptual design progresses, the coefficients of the linearized design criterion should be reassessed.

## ACTIVE CONTROL ANALYSIS PACKAGE

The collection of computer programs that embodies the active control design approach being proposed here is called the Active Control Analysis

Package. The major groupings shown in figure 8 are sizing, modeling, control, response, and adjust modules. The design philosophy is to represent the interests and constraints of all participants by mathematical relationships as compared to human participants (ref. 9). Implementation of such a notion is still in its infancy and much more work remains to be done before the goals of the Active Control Analysis Package can be achieved. A major contribution to ACAP is expected to be a portion of the program being developed by Aerospace Systems Incorporated (ref. 10).

### Sizing Module

This part of the program involves aerodynamic, control, and response modules. The aircraft sizing module of the analysis package consists of little more than the fundamental relationships contained in every conceptual design of a subsonic civil transport. The only new thing is that the procedure has been computerized and a definitive criterion is maximized automatically. Provisions have also been made to include the effects of active control functions. A technique for estimating the costs and weights of active controls is being developed by Boeing (Contract NAS1-14064). The results of this work are incorporated in the sizing module. The sizing module continues to be developed, but already it has proved to be useful. The aircraft sizing module was the means by which the previously discussed linearized design criterion was determined. It has also been used to assess the benefits of active controls, the supercritical wing, composite structures, and laminar flow control. Figure 9 shows the results of the sizing program for airplane configurations that either maximize ROI or seat miles per gallon. The incidental improvement in seat miles per gallon due to the supercritical wing and active controls is about 6 and 12 percent, respectively. The ground rules used to compute the comparison are given in table I.

### Modeling Module

The dynamic modeling portion of the Active Control Analysis Package includes making linear constant coefficient state space models that include static and unsteady aerodynamics, structural dynamics, sensor dynamics, and control compensation into a single equation of the form  $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$ . The streamlined doublet-lattice technique is presently being used for the unsteady aerodynamics with an alternative module using Morino aerodynamics (ref. 11) soon to be available. Comparisons of these methods for determining the unsteady aerodynamic characteristics are about to start. The advantage of having the unsteady aerodynamics in the state variable format is that it enables a single mathematical model to be used for analysis of both high-frequency phenomenon such as flutter and low-frequency "rigid-body" motion. A control law can thus be synthesized without having to make a special investigation of flutter stability. Figure 10 shows an example of a root locus of dynamic pressure, where the critical dynamic pressure for flutter was accurately determined by the state variable model. The results that are almost identical to those obtained by frequency domain techniques indicate the validity of the state variable model at high frequencies. The dynamic

model was obtained using a technique pioneered by personnel at the Boeing Airplane Company. The results shown in figure 10 use programs developed at the Langley Research Center by Irving Abel and Ray Hood. The specific calculations were performed by Mr. Hood.

### Control Module

The approach taken to synthesize the various active control laws are outlined in figure 11. For maneuver load control, feedforward compensation will be used to the extent possible. The purpose is to minimize primarily the wing weight by reducing wing bending moment due to the 2-1/2 g maneuver required of civil transport aircraft. By employing feedforward, the stability of the airplane is not affected and numerous feedback problems are avoided. Only in the event of considerable uncertainty of the plant characteristics will feedback be used. For gust load alleviation and ride quality control, the approach is to measure the gust as accurately as possible and feed this forward as well. There will be a frequency range (probably a few Hertz) beyond which the accuracy of the measured gusts will be too poor to be of any use. At these higher frequencies, it will be necessary to desensitize the aircraft response through feedback designed for this purpose. Feedback synthesis presents a particularly difficult challenge, however. Because of the extremely large number of state variables involved (fig. 12), reduced state feedback is necessary. A couple of approaches to synthesizing reduced state control laws are being pursued. Figure 13 is a diagram in which stochastic models of both disturbances and desired responses are used to formulate a quadratic minimization problem. An algorithm for solving this problem for constrained feedback and feedforward matrices is being developed by Dr. Howard Kaufman of Rensselaer Polytechnic Institute (NASA Grant NSG-1188). An alternate approach is being developed by Dr. Isaac Horowitz, University of Colorado (NASA Grant NSG-1140). Dr. Horowitz has developed a computer program that solves the difficult synthesis problem which minimizes the bandpass for given levels of uncertainty in the plant parameters and a particular system performance requirement. It is hoped that this solution for the single input/single output problem will serve as a building block for more complex practical problems. A quadratic cost function will be used involving the mean-square acceleration at various stations of the aircraft. For reduced state feedback, a quadratic cost function based on pilot ratings and passenger comfort will be employed. Figure 14 shows some preliminary results in expressing pilot ratings as a quadratic performance index suitable for control synthesis. Similar results have been obtained for the pitch degrees of freedom. The work is being done by Dave Middleton of Langley Research Center. Reduced state feedback will be required because of the complexity of the aeroelastic modes. Flutter suppression and structural mode control will include both aeroelastic modes and unsteady aerodynamics in models used for control synthesis to assure stability and long structural life. All systems are considered at various levels of complexity and in several flight and loading conditions. The linearized design criteria will be used to assess candidate system configurations considering not only their system performance but their reliability and maintainability as well.

## Response and Adjust Modules

The response module computes the response of all aspects of the airplane being considered. These responses are compared with constraints not contained in the sizing or other modules. Adjustments are then made based on this comparison. This loop is iterated until convergence is obtained, resulting in a design that satisfies all constraints and maximizes the design criterion.

## CONCLUDING REMARKS

The problem of incorporating active controls in the conceptual design of a civil transport is a very complicated and involved problem. It is a problem that requires our best understanding and sophisticated computer programs to assist us in the very complex trade studies required. The purpose of the NASA Active Control Technology Program is to contribute to the solution of this task by developing an Active Control Analysis Package. The package will not only add to the analysis tools available but will organize them to greatly facilitate conceptual design studies.

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TABLE I.- PARAMETER VALUES USED IN COMPARISON OF TECHNOLOGY BENEFITS

	Wave drag parameter	Trim drag	Skin friction	Tail area	Maintenance costs	Strength/Wt	Ult. load
	$\frac{t}{c}(0)$	$C_{DTRIM}$	$C_{DOWING}$	$\frac{S_{HT}+S_{VT}}{S_{WING}}$	$\frac{MAINTENANCE}{BLOCK HOUR}$	$K_{BS}$	g's
Conventional	0.88	0.0007	0.0070	0.35	\$ 0	86,800	3.75
Supercritical	.92	.0007	.0070	.35	0	86,800	3.75
Active controls	.92	.0000	.0070	.21	10	86,800	3.0

Class of Airplane Studied:

Payload — 300 passengers, max

Range — 3750 n. mi.

Cruise Mach number — allowed to vary. M 0.84

Design criteria — ROI or SMPG

Payload factor — 55%

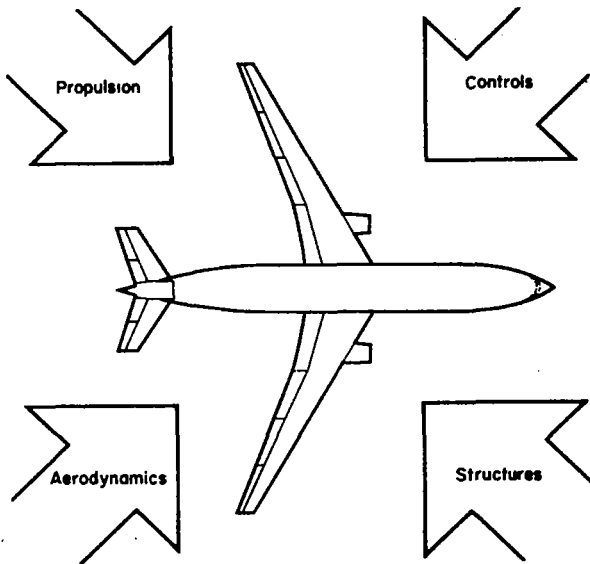


Figure 1.- The job of integrating propulsion, aerodynamics, structures, and controls in a conceptual design.

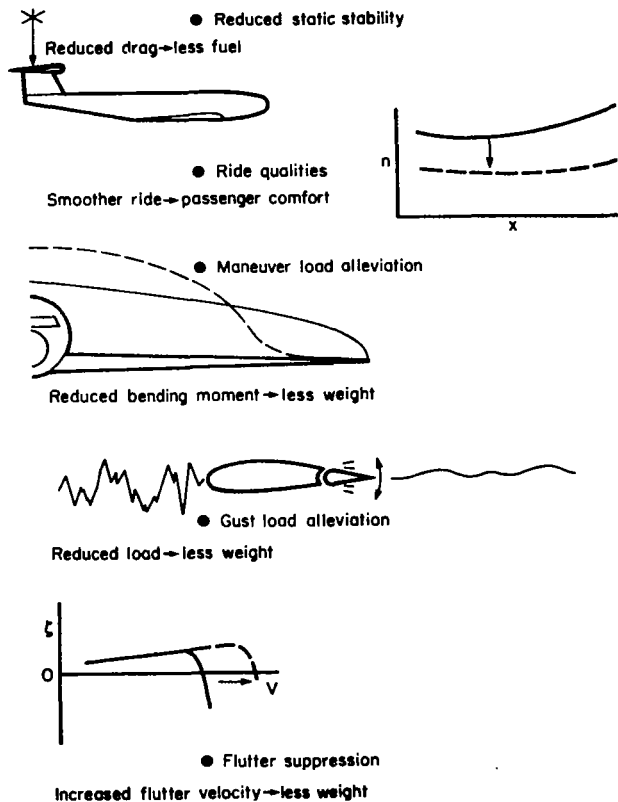


Figure 3.- Active control concepts.

- Reduced static stability - less trim drag
- Gust load alleviation - less structural weight
- Maneuver load alleviation - less structural weight
- Ride qualities - passenger comfort, longer fatigue life
- Flutter suppression - less structural weight, longer fatigue life
- Envelope limiting - flight safety
- Decoupled control - improved handling qualities

Figure 2.- Active controls.

#### Objective

"The overall objective of the active control technology program is to identify, develop, and validate the controls technology required for the integration of active controls in future subsonic civil aircraft."

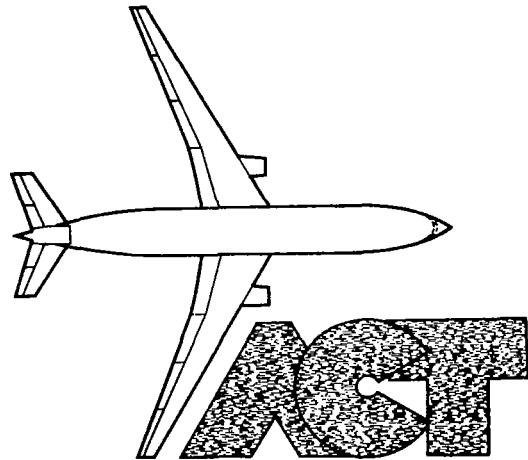


Figure 4.- Objective of the active control technology program.

1. Baseline data
2. Validation flight and wind tunnel tests
3. Active control analysis package
4. Integrated conceptual designs

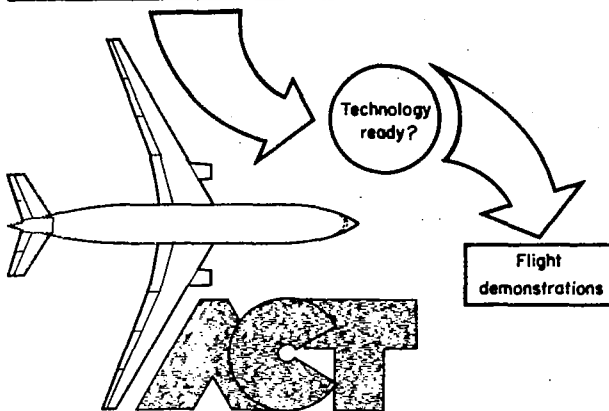


Figure 5.- Overall active control aircraft program.

	Aero	Propulsion	Structures	Controls
Performance	x	x	x	x
Cost			x	x
Revenue	x			
Safety		x	x	x
Noise		x		
Pollution		x		
Fuel consumption	x	x	x	x

Return on investment

$$\begin{pmatrix} \text{Wing area} \\ \text{gross weight} \\ \vdots \end{pmatrix} = C^* = \underset{C}{\text{ARG max}} (ROI)$$

$$f(c) = 0$$

$$g(c) \leq 0$$

Figure 6.- Design criterion.

$$\Delta ROI\% = \frac{\partial ROI}{\partial \text{weight}} \Delta \text{weight} + \frac{\partial ROI}{\partial \text{initial cost}} \Delta \text{initial cost} + \frac{\partial ROI}{\partial \text{operating cost}} \Delta \text{operating cost}$$

$$= \frac{\Delta \text{weight}}{(5657 \text{ lb})} + \frac{\Delta \text{initial cost}}{(\$10 \text{ M})} + \frac{\Delta \text{operating cost}}{(\$126/\text{hr})}$$

$$\Delta \text{weight cost} = \Delta \text{weight, structure} + \Delta \text{weight, system}$$

$$\Delta \text{initial cost} = \Delta \text{initial cost, structure} + \Delta \text{initial cost, system}$$

$$\Delta \text{operating cost} = \Delta \text{maintenance cost} + \Delta \text{operating cost, liability (MTBF, pilot rating)}$$

Figure 7.- Linearized design criterion.

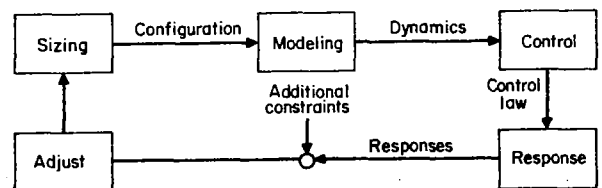


Figure 8.- Active control analysis package.

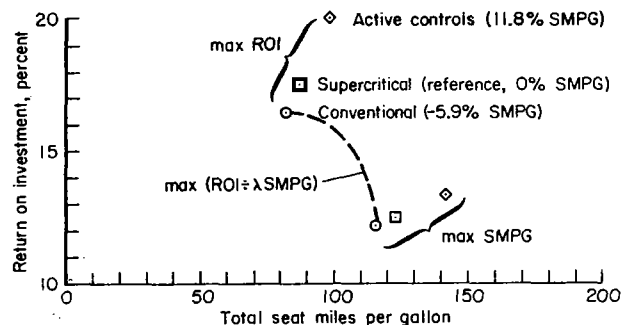


Figure 9.- Return on investment and total seat miles per gallon for various designs.



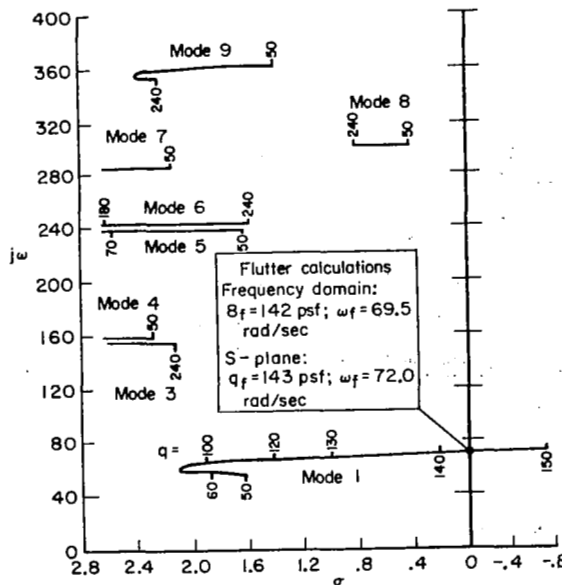


Figure 10.- Dynamic pressure locus using state space aeroelastic model.

Maneuver load control	Feedforward, minimize primarily wing weight by reducing wing bending moment due to 2.5 g maneuver
Gust load alleviation, ride control	Measure gust, feedforward, desensitize the aircraft response through feedback designed using reduced state, quadratic cost algorithm
Reduced static stability	Optimal reduced state feedback using quadratic cost function based on pilot ratings
Flutter suppression structural mode damping	Include aeroelastic modes unsteady aerodynamics in models used for control synthesis to assure stability
All systems	Consider various levels of complexity and at several flight and loading conditions. Use linearized design criterion to assess candidate system configurations

Figure 11.- Approach taken in synthesizing active control laws.

# STATE VARIABLES FOR LONGITUDINAL, SYMMETRICAL MODES:

	No. of states
Rigid body modes ( $\alpha, q, \theta, V$ )	4
Aeroelastic modes (9 second order modes)	18
Unsteady flow (+2 each mode, control)	26
Actuator dynamics	8
Sensor dynamics	10
Filters, compensators	<u>8</u>
TOTAL	74

If all modes are coupled because of engine momentum or malfunctioning actuator, the total states number is about 150.

Conclusion:

Need reduced state, insensitive control, synthesis techniques, and algorithms.

Figure 12.- The problem of too many state variables.

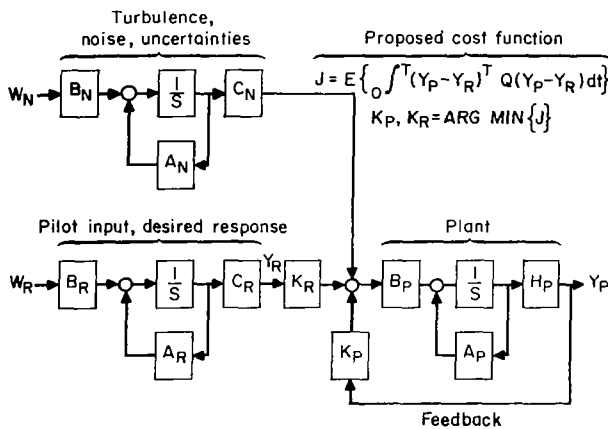


Figure 13.- Control law synthesis using reduced state feedback.

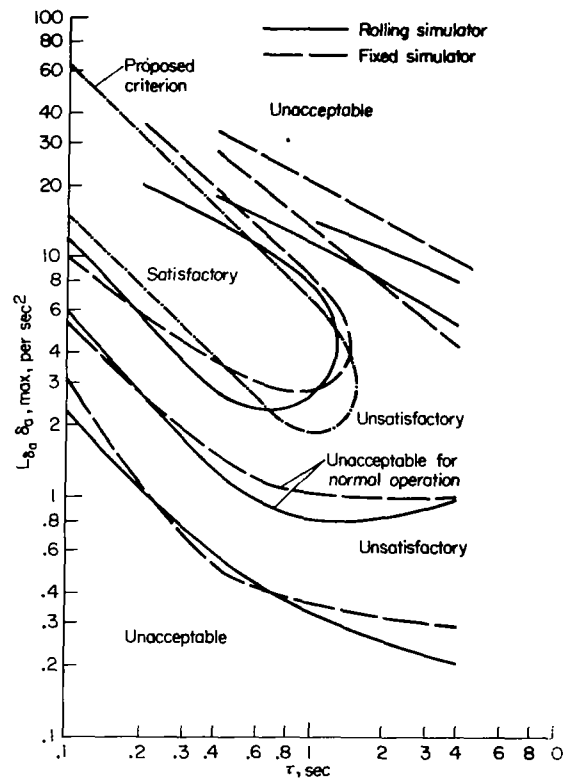


Figure 14.- Comparison of pilot opinion boundaries.

## NASA AMES ACTIVE CONTROL AIRCRAFT FLIGHT EXPERIMENTS (ACA) PROGRAM

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The objectives of the Ames ACA program are to develop the active control technology (ACT) for short-haul aircraft, to evaluate existing methods, to develop new techniques, and to demonstrate the readiness of the technology in operational environment. Two concepts are basic to ACT: integrated aircraft design and integrated flight-control-system design. These two concepts form the basis of the Ames program.

Studies have shown that significant reductions in induced drag and weight, improvement in passenger comfort, and reduction of flight hazards can be achieved with ACT. These benefits are possible due to (a) a reduction in the size of stabilizing surfaces, with stability provided by dynamically controlling movable surfaces (reduced static stability design) rather than statically with larger fixed surfaces as in conventional design; (b) reductions in structural strength requirements by applying maneuver load and gust load alleviation and flutter mode control; (c) improvement of ride by a ride quality control system; and (d) reduction in the occurrence of inadvertent flight hazard through automatic limitation of flight condition. The benefits are maximized when ACT is factored in the design and integrated with aerodynamics, structures, and propulsion early in the design cycle of the aircraft. But, before such an ACT-configured aircraft can be designed with confidence, existing mathematical modeling techniques must be evaluated. In comparison with the conventional flight-control-system design, the design of such ACT systems as flutter mode control and reduced static stability requires significantly more accurate mathematical models of the process to be controlled. Of particular concern are the models of unsteady aerodynamics of the basic aircraft, control surfaces, and sensors in the flow field; models of flexible structures of the airframe, control surfaces, effects of flexibility on sensors and actuator linkages, and aeroelastic coupling; and the models of propulsion systems, including engine power dynamics and control, and power-aerodynamics coupling, the latter being particularly significant in powered lift configurations. Therefore, a part of the Ames program is directed at the evaluation of the complete aircraft modeling methodology.

The process resulting in an optimum ACT-configured design is expected to require many iterations on the aircraft geometry, including size and location of control surfaces. At each such iteration, the effects of the control system must be determined, thus requiring a mathematical model of the controlled process. Since the availability of efficient algorithms for transforming aircraft configurations into mathematical models for control-system design purposes is implicit in the ACT design approach, particular attention is given in the Ames program to digital computation methods based on potential flow theory (ref. 1). Experiments are being planned to evaluate

such methods by comparison with flight-test data. The methodology and algorithms of system identification are expected to play a prominent role in extracting model parameters from flight-test data.

The second basic concept of Active Control Technology is that maximum benefit is obtained from automatics when all functions are blended into a single, fully integrated flight-control system. A good working hypothesis is that the next generation of flight-control systems will provide four sets of functions: (1) ACT modes, including gust load alleviation, maneuver load control, flutter mode control, and ride quality control, all of which improve aircraft performance without adversely affecting the pilot handling qualities; (2) stability and control augmentation system (SCAS), a set that provides an interface between the pilot and the basic aircraft dynamics, such as the stabilization of an aircraft with reduced static stability; (3) full-flight envelope autopilot, a set whose elements provide the pilot with a variable degree of automation, from SCAS (requiring maximum pilot participation) to completely automatic four-dimensional trajectory tracking, automatic envelope limiting, and configuration management; and (4) automatic fault regulator, which provides automatic fault detection, fault identification, and system reconfiguration to bypass the fault.

Before such an integrated flight-control system can be implemented with confidence, several system-theoretic problems of varying degrees of difficulty must be solved. Powerful concepts (ref. 2) and computerized algorithms exist for the design of linear systems and can often be applied to the design of ACT modes and SCAS, which are primarily perturbation regulators so that linearity and even time invariance are reasonably valid. However, as already noted, there may be significant uncertainties in the actual plant parameters and plant structure. Methods for the reduction of input-output sensitivity to perturbations in the plant are needed. Linear methods are also helpful for the design of autopilots, but to a much lesser degree. Increased trajectory tracking accuracy requirements of projected STOL and VTOL operations, coupled with inherently nonlinear force and moment generation processes in powered-lift aircraft, make it very difficult to cover the operational envelope of such aircraft with a set of linear models and control laws linked by a gain schedule. Alternate methods for coping with the nonlinearities must be developed.

The wide range of lift and drag coefficients required in short-haul operation is achieved by in-flight modification of the aircraft configuration (ref. 3). Inside the flight envelope there are, normally, infinitely many combinations of controls which result in identical lift and drag coefficients — that is, given values of lift and drag define a surface rather than a single point in the control space. The remaining freedom can be used to optimize system performance and increase both safety margin and aircraft maneuverability. The methodology for automatic configuration optimization and envelope limiting and for the integration within the autopilot of ACT modes and SCAS must be developed. A part of the Ames program is directed at the development and evaluation of such a methodology.

Normal operation of all system components was tacitly assumed in the discussion thus far. When the possibility of component failures is introduced into the synthesis, the available synthesis methodology loses most of its power. There is a pressing need for major advances in this area, particularly in the development of a theoretical foundation for the design of fault regulators, which detect and identify failures and reconfigure the control logic into the next safest mode of operation. The need for fault regulators is evident in the case of advanced flight-control systems with high authority, flight critical functions. However, it would seem that the majority of control systems in the future, regardless of the particular field of application, will be controlling with high, if not complete, authority complex, critical systems and therefore will have to regulate faults automatically. Thus, advances to be made in flight-control systems will very likely have wide-ranging applications.

A part of the Ames program is directed at the development and evaluation of the fault regulation methodology. The scope is limited to (1) synthesis of the reconfiguration logic, given a complete description of every failure from a given set; (2) sensor subsystems that provide, through hardware redundancy and functional redundancy, state estimates and their accuracy including extreme failures; and (3) actuator subsystems that provide, through hardware redundancy, estimates of command execution accuracy. Other topics, such as hardware failures in the flight computer, are excluded from the program.

All three parts of the Ames program, namely, modeling methodology, integrated flight-control-system design assuming no component failures, and fault regulation methodology will be developed and evaluated by generalization from particular applications to specific research aircraft. In the first phase of the program, research will be focused on the DHC-6 (Twin Otter) aircraft — an example of a low wing loading configuration with conventional lift. The aircraft has been modified to include a set of spoilers for direct lift, drag, and roll control. In addition, the aircraft has been equipped with a STOLAND system, including a flight computer with access to a large set of sensors and actuators. In the second phase, the research will be focused on powered-lift configurations similarly equipped with functionally powerful hardware. Candidates for that phase are XV-15 (tilt-rotor) research aircraft and possibly either YC-14 or YC-15 (advanced medium STOL transport).

The first major objective of the program is to formulate a complete overall logical structure for automatic flight-control systems that can (1) admit any combination of ACT modes; (2) provide accurate, automatic coupling to air traffic control, which may select any trajectory from a large set of complex trajectories; (3) provide a spectrum of modes of pilot-automatic interaction; (4) accept a variable sensor set of variable accuracy and a variable actuator set; (5) accept a highly nonlinear plant with functionally redundant controls; and (6) provide a framework for the design of fault regulators. The logical structure being developed at Ames is described briefly here. (A more detailed description is given in ref. 4.)

The structure of the control logic is hierarchical in the sense that there is a nesting of subsystems. At each level in the hierarchy, relatively

simple commands are accepted from above and transformed into more detailed commands downward. The command is based on a relatively simple input-output model of lower levels, and the levels have a degree of autonomy. Figure 1 shows the control logic as viewed from the level of the trajectory control system. At the top of the hierarchy is the air traffic control (suitably generalized to include the pilot) which requests a trajectory that may or may not be executable from the existing state of the aircraft. The trajectory command generator transforms this request into an executable open-loop acceleration command and a corresponding four-dimensional trajectory that may, depending on the original request, have ignorable parameters. The acceleration command is sent downward in the hierarchy to the trajectory acceleration controller, which is expected to execute that command in accordance with a relatively simple input-output relation and error bounds. Errors arising from wind turbulence, uncertainties in plant parameters, intentional oversimplification of the dynamics, etc., are controlled by means of the trajectory perturbation controller in which estimated aircraft position, velocity, and acceleration are compared with the command, and a corrective acceleration generated and added to the open-loop command, thereby closing the loop around the errors.

Figure 2 shows the structure of the trajectory acceleration controller for the case in which lift is controlled by angle of attack and side force is controlled by the roll angle, so that aircraft attitude becomes an indirect control variable. The closed-loop acceleration command enters the force trimmap, an algebraic (static) representation of the total (aerodynamic and propulsive) force generation process. At this level, configuration management and envelope limiting take place. The output is the attitude and remaining controls which, in the absence of errors, would trim the aircraft to the required acceleration. The trim attitude, possibly time-varying, is sent to the attitude control system, which is similar to the trajectory control system. The trim attitude is transformed in the rotation command generator into an executable open-loop angular acceleration command and a corresponding angular velocity and attitude time histories. The angular acceleration command is sent down the hierarchy to the angular acceleration controller, which is expected to execute the command with a relatively simple input-output relation and error bounds. Attitude errors are controlled by means of the attitude perturbation controller in which estimates of aircraft attitude angular velocity and angular acceleration are compared with the command, and a corrective angular acceleration is added to the command, thereby closing the loop around the errors. In figure 2, the angular acceleration controller is represented by the moment trimmap. The control surface servos, at the bottom of the hierarchy, are not shown.

The proposed logical structure has several advantages, among which are the following. It is applicable to a large class of aircraft. The particulars of the aircraft affect primarily the force and moment trimmaps. The design approach is nearly algorithmic. Performance, sensor accuracy, and actuator accuracy enter as independent variables that may be varied over a wide range. There is an effective tradeoff between performance and flight computer requirements. Because the control logic is hierarchical, problems

associated with the design of fault regulators are considerably simplified. A flight-test evaluation of the control logic is expected to occur in the summer of 1976.

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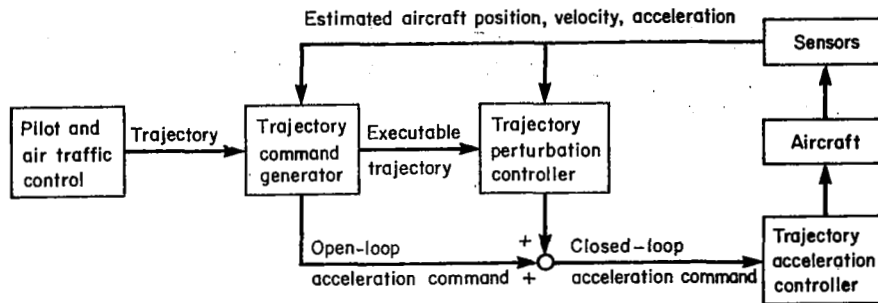


Figure 1.- Trajectory control system.

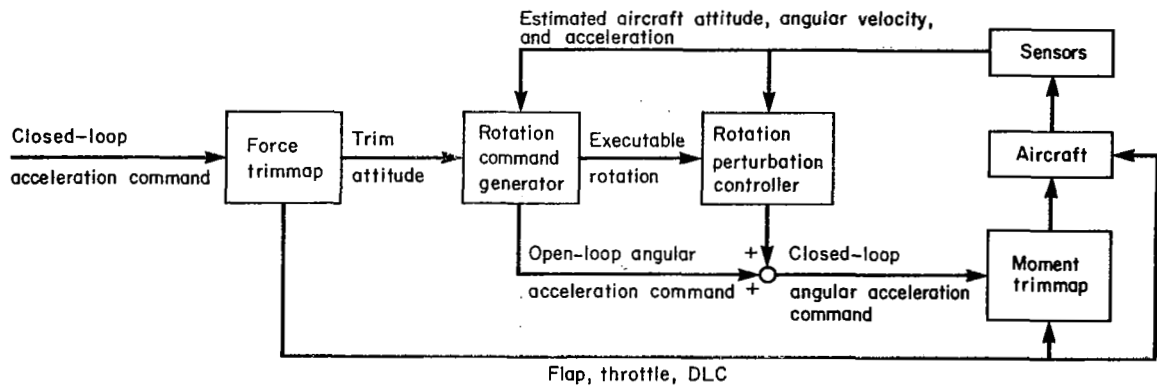


Figure 2.- Trajectory acceleration controller.



# ADAPTIVE CONTROL LAWS FOR F-8 FLIGHT TEST\*

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## SUMMARY

This paper describes an adaptive flight-control-system design for NASA's F-8 digital fly-by-wire research aircraft. This design implements an explicit parallel maximum likelihood identification algorithm to estimate key aircraft parameters. The estimates are used to compute gains in simplified quadratic-optimal command augmentation control laws. Design details for the control laws and identifier are presented, and performance evaluation results from NASA Langley's F-8 Simulator are summarized.

## SECTION I - INTRODUCTION

NASA is presently conducting research in digital fly-by-wire flight-control technology. In the first phase of this research, an F-8C test aircraft was modified to use an onboard Apollo guidance computer for flight-control calculations. The computer was connected directly to the aircraft's electrohydraulic actuation systems, with no mechanical linkages remaining between pilot stick and control surfaces. This configuration was flight tested to demonstrate the practicality of digital fly-by-wire control in an actual test vehicle (ref. 1).

For the second phase of the research program, the test aircraft is being outfitted with triply redundant AP-101 computers, which are again connected directly to (improved) electrohydraulic actuation systems. This new configuration serves as a test bed for studies and demonstrations of redundancy concepts to enhance reliability and advanced control laws to improve the performance and/or overall effectiveness of the aircraft. For control law studies, in particular, the new computer's speed, memory size, and floating-point capability removes most of the complexity constraints imposed by more typical flight-control machines.

To date, two sets of control laws have been scheduled for flight experimentation in the new F-8 configuration — a "CCV package" and an "adaptive package." The first package is described in reference 2. It provides basic

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command augmentation functions, outer loops, and control modes for various control-configured-vehicle (CCV) concepts applicable to fighter aircraft. The second control law package is described here with further details in reference 3. It consists of the same command augmentation functions found in the CCV package, but adds explicit on-line maximum likelihood identification capability for adaptive control. In addition to these packages, research has also proceeded on several other candidate adaptive algorithms for the F-8 (refs. 4-6) and on sensor fault detection and isolation algorithms (refs. 7 and 8). Some of these will reach flight experimentation later in the program.

The purpose of this paper is to describe the contents of the Adaptive Control Law package. We begin the description (section 2) with a discussion of general design objectives and ground rules. We then introduce the overall adaptive system structure and discuss the design of its major components - the control law (section 3.2) and the explicit identifier (section 3.3), followed by simulator performance evaluations in section 4 and by a summary of remaining issues and plans in section 5. Of necessity, many of the modeling, design, and evaluation details cannot be covered here. For these details, the interested reader is referred to references 2 and 3.

## SECTION II - OBJECTIVES AND GROUND RULES

NASA's primary reasons for studying adaptive control laws for the F-8 aircraft are to identify potential performance and system effectiveness benefits offered by modern adaptation/identification concepts and to demonstrate these in an actual digital flight-control environment. Hence, the aim of the design effort was not to develop new theoretical procedures or algorithms but to turn existing concepts into flightworthy control laws for the specific test aircraft. Of course, the concepts and design processes should be general enough to apply to other aircraft as well. This is important because the F-8C itself is not a state-of-the-art fighter, nor is it difficult to control with standard nonadaptive techniques. Other more specific groundrules imposed on the design include the following.

### Inputs

The adaptive system must operate in the presence of normal pilot inputs and also when such inputs are absent. Any test signals required for the latter case must be small enough not to interfere with the aircraft's mission. This generally means that test input normal accelerations, as sensed at the pilot station, should be below 0.02-0.03 g rms, and lateral acceleration should be even lower in the range of 0.01 g. This ground rule establishes a crucial distinction between identifiers designed for operational adaptive controls and those designed for postflight-test data-processing applications (refs. 9 and 10). In the latter case, test inputs are deliberately large and often optimized for identification accuracy (refs. 11 and 12). For operational adaptive controls, the emphasis must be to make these inputs small and hopefully unnoticeable.

## Sensors

The system was constrained to operate with the aircraft's existing surface (servo) position transducers and inertial sensors (rate gyros and accelerometers), but without attitude measurements and without air data measurements (dynamic pressure, velocity, angle of attack). These limitations are imposed by the sensor complement available on the test aircraft and, in the case of air data, by the philosophical bent of the authors. We believe that removal of flight-critical air data measurements is one of the most tangible benefits adaptive controls can offer for aircraft (like the F-8) whose basic performance requirements are readily satisfied with air-data-scheduled control laws.<sup>1</sup>

## Computer Capacity and Sample Rate

Since control law calculations typically consume only a small fraction of the total computational load in a flight computer (the bulk is I/O, self test, mode and redundancy control), the adaptive law's goals are restricted to 25 percent of the available frame time of the AP-101 and to a "reasonable" allocation of memory. The sample rate of the computer was prespecified to be 50 Hz. This gives 5 msec real time per sample in which to complete all control and identification calculations. The sample rate is also high enough to produce no substantial differences between direct digital design (discrete-time control laws designed for discrete-time models) and continuous time design with after-the-fact discretization.

## Control Surfaces and Actuators

The standard elevator, aileron, and rudder surfaces were available for adaptive control. Leading-edge and trailing-edge flaps were assumed to remain undeflected or to follow their open-loop scheduled minimum drag positions. Actuation systems for these surfaces were represented by models incorporating projected characteristics of the improved systems being installed on the test aircraft.

## SECTION III - ADAPTIVE SYSTEM DESIGN

The overall structure chosen for the F-8's adaptive control system is based on explicit separation of identification and control (fig. 1). A standard gain-scheduled control law was designed for the aircraft's complete flight envelope and the scheduling functions (gain adjustment rules) are

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<sup>1</sup>In flight-critical digital fly-by-wire control systems, reliability requirements call for triply redundant or even quadredundant air data sensors. Aside from the cost and complication of these multiple sensors, suitable probe locations impose difficult compromises between data quality, channel tracking, and dispersion to avoid common hazards.

supplied from a separate aircraft identifier. While such *a priori* separation is known to be theoretically imperfect in the dual-control sense (ref. 13), it is ideally suited, both historically and in terms of practical features of the solution, to the flight-control problem. This will become evident as we describe aircraft models and features of the separate controller and identifier designs.

### Aircraft Models

Mathematical models for the F-8C were developed from general six-degree-of-freedom nonlinear equations of motion together with geometric data, mass properties, and aerodynamic data tables (ref. 14). These equations were linearized numerically at 25 flight conditions throughout the operating envelope of the aircraft. Each flight condition is defined by the nominal altitude, Mach number, weight, geometric configuration, and load factor which characterize the models obtained by the linearization, that is,

$$\left. \begin{aligned} \dot{x} &= F(\underline{c}_i)x + G_1(\underline{c}_i)\delta + G_2(\underline{c}_i)w + G_3(\underline{c}_i) \\ y &= H(\underline{c}_i)x + D(\underline{c}_i)\delta, \quad i = 1, 2, \dots, 25 \end{aligned} \right\} \quad (1)$$

where  $x$  denotes 12 aircraft states,  $\delta$  denotes 3 control surface deflections,  $w$  denotes wind gust disturbances, and  $y$  denotes assorted outputs we wish to measure and/or control. The matrices  $F$ ,  $G_1$ ,  $G_2$ ,  $G_3$ ,  $H$ , and  $D$  can be treated as continuous functions of the slowly changing nominals  $\underline{c}$ , with values known at discrete points.

Equation (1) is not usually used directly for control design. It is first partitioned into longitudinal and lateral-directional degrees of freedom, and each partition is separated into dynamic modes that dominate the frequency range of interest in the design task (ref. 15). The resulting reduced models are then augmented with actuator, sensor, gust, and flexure characteristics as necessary to make up the complete design model.

For the F-8C adaptive system, the control objective was to provide basic stability and command augmentation functions. The reduced models that suffice for this objective consist of second-order short-period dynamics plus a first-order power actuator for the longitudinal axes. A fourth-order dutch roll, roll subsidence, and spiral mode model suffices for the lateral-directional axes. The longitudinal model is summarized in figure 2 and table I, and the lateral-directional model can be found in reference 2.

Figure 2 and table I provide model equations and approximate functional descriptions for the equation coefficients, respectively. The latter functions were derived by plotting numerical values from the 25 flight conditions against two dominant nominal flight condition parameters, dynamic pressure ( $\bar{q}$ ), and Mach number. The trends in these plots were approximated by simple functions, and the scatter was represented by additional small-perturbation parameters with bounded uncertainties. Hence all coefficients have the form

$$p_i = f(\bar{q}, \text{Mach}, c_i) \quad (2)$$

or the form

$$p_i = g(M_{\delta o}, \text{Mach}, c_i) \quad (3)$$

where equation (3) is obtained by substituting the very good linear relationship between surface effectiveness ( $M_{\delta o}$ ) and  $\bar{q}$  into equation (2). The point of including these model data here is to show how strongly the parameter  $M_{\delta o}$  (or equivalently  $\bar{q}$ ) dominates the F-8C's dynamics. Consequently, this same parameter also plays a dominant role in the control law and identifier designs to follow.

### Control Law Design

Both the longitudinal and lateral-directional control laws were designed with modern LQG techniques (ref. 16) appropriately tempered with established classical design requirements. In each case, a quadratic optimization problem was formulated and solved at selected flight conditions under assumptions of noiseless, full-state, continuous-time feedback. The resulting control gains and dynamics were then simplified and approximated as functions of measurable nominal-flight condition parameters to provide a standard gain-scheduled control law. We describe this design process in more detail below for the longitudinal control law only. The lateral-directional details are again provided in reference 2.

*Longitudinal optimization problem-* To gain ready acceptance as flight-worthy, the F-8C's longitudinal control laws were designed to satisfy the following list of classical performance specifications:

1. Command augmentation functions

- a. C\* step response
- b. Stick gradient to meet MIL-F-8785 (ref. 17)
- c. Neutral speed stability

2. Regulator function

Reasonably damped gust responses (short-period damping ratio > 0.3)

3. Tolerance for uncertainties

- a. Classical loop gain and phase margins ( $GM > 6$  db,  $\phi > 30^\circ$ )
- b. Loop attenuation at high frequencies ( $< -20$  db at 8 Hz)

These requirements are obviously not in the quadratic cost function form required by LQG theory. Hence an important step in the design process was to reinterpret them in more suitable terms.

Consider the command augmentation requirements first. Item (1a) in this group is a dynamic response specification on  $C^*$ , a linear combination of pitch rate and normal acceleration:

$$C^* = n_z + V_{co} q, \quad V_{co} = 324 \quad (4)$$

In response to step commands from the pilot's control stick, this variable must stay within a specified response envelope (ref. 18) whose center is closely approximated by the step response of a second-order linear model with  $\omega \cong 7$  rad/sec,  $\zeta = 0.9$ . This requirement was incorporated in the LQG framework by the common technique of appending an explicit "command model" and penalizing errors between it and the aircraft (refs. 19 and 20). Specification item (1b) was then used to set the DC gain (acceleration per unit stick force) of the model, and item (1c) was satisfied by penalizing an appended integral of the model-following error as well as the error itself. The latter approach is motivated by the fact that "neutral speed stability" is actually imposed to provide command insensitivity to trim changes. A pilot should not be obligated to readjust trim or to provide compensating stick force inputs as the aircraft changes speed (and hence as it changes the trim disturbance terms  $M_0$  and  $g/V$  in fig. 2). A bit of integral control action provides this desired insensitivity.<sup>2</sup>

In addition to the command model and integrator appended to satisfy items (1a) to (1c), it was also necessary to append a low-pass filter on the normal acceleration signal used to construct  $C^*$ . This filter provides high-frequency attenuation, item (3b), which is required to assure adequate stability margins in the presence of uncertain servo characteristics and unmodelled flexure dynamics of the airframe. Loop transfer functions of trial designs without such filters tended to drop off too slowly to meet the specification. Slow roll-off is, of course, a general property of quadratic-optimal control laws, formally recognized but frequently forgotten (ref. 21).

The complete optimization problem with these appended states is illustrated in figure 3. As shown, the criterion function was taken as a weighted sum of four quadratic terms — model-following error ( $C^* - C^*_c$ ), integrated model-following error  $\int (C^* - C^*_c) dt$ , elevator rate  $(d/dt)\delta_e$ , and elevator command  $\delta_c$ . Quadratic weights were then selected iteratively until the resulting control law satisfied all performance specifications. Representative weights and gains are summarized in table II.

*Control law simplification*— As indicated earlier, the quadratic optimal control law was next simplified and approximated as a function of nominal flight condition parameters. While several numerical algorithms have been developed to aid in these reductions (refs. 20, 22, and 23), the F-8C's longitudinal control law was compact enough to be reduced by standard block diagram manipulations, cancellation of nearly equal poles and zeros, and removal of low-gain paths. The result is shown in figure 4. It consists of a basic proportional-plus-integral  $C^*$  feedback loop, modified slightly by the

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<sup>2</sup>Neutral speed stability is desirable during all flight phases except power approach. For the latter, the integral action must be switched out.

appended lag network on measured acceleration and by a lead-lag network on pitch rate. The pilot's stick force commands this loop through a feedforward filter that consists of the original  $C^*$  model with altered numerator dynamics. All parameters in the control law are independent of flight condition except the loop gain,  $G_{C^*}$ , which varies inversely with surface effectiveness (or dynamic pressure). It becomes desirable to adjust other gains only if the aircraft is flown with deliberately relaxed static stability.<sup>3</sup> For such conditions, the dotted line path in figure 4 must be added to the control law, with gain  $G_{RSS}$  adjusted as a function of  $M_{\delta_0}$  and  $M_{\alpha}$  (or  $\bar{q}$  and Mach).

Numerical values for the gain  $G_{C^*}$  are shown in figure 5, where they are also compared with the critical gain at which the (linear) loop reaches instability. It is clear from these curves that the F-8C is a very tolerant aircraft.  $G_{C^*}$  easily achieves 6 db of margin over the entire flight envelope, and even a fixed gain system could fly the aircraft adequately at all but the lowest dynamic pressure conditions. This fact deserves emphasis because it complicates evaluations of an adaptive system designed for this aircraft. Total closed-loop performance does not provide a sensitive measure of adaptive capability, and other less direct criteria are required to evaluate the system.

*Lateral-directional control laws*- Although details are left to references 2 and 3, the lateral-directional control design process resulted in structures very similar to figure 4. The roll channel in figure 6 is a simple roll-rate feedback loop commanded through a first-order model, while the yaw channel in the figure combines compensated yaw rate, roll rate, lateral acceleration, and aileron position feedbacks with direct rudder pedal commands. Both loops have one gain that changes with flight condition in inverse relation to dynamic pressure and hence in direct proportion to gain  $G_{C^*}$ .

### Identifier Design

In nonadaptive flight-control implementations, variable gains like those in figures 5 and 6 are usually adjusted with external measurements obtained from the aircraft's air data system. This approach will be used in the F-8C's first flight tests of the CCV Control Law Package planned for mid-1976. As already noted, however, air data systems can be troublesome components in the overall flight-control mechanization, and it would be very desirable not to need them. The identifier design is intended to accomplish this end. Its primary function is to provide an adequate surface effectiveness estimate ( $\hat{M}_{\delta_0}$ ) to substitute for measured  $\bar{q}$ . It also estimates several other quantities - pitching moment due to angle of attack ( $\hat{M}_{\alpha}$ ), true airspeed ( $\hat{V}$ ), and total angle of attack ( $\hat{\alpha}$ ). These are sometimes needed to schedule more complex control laws than those that suffice for the F-8.

<sup>3</sup>This is a potential CCV feature achieved by large aft displacements of the c.g. Its benefits are reduced trim drag at the expense of basic airframe stability. In the model coefficients of table I, it means replacing  $M_{\alpha}$  with the functions  $c_2 M_{\delta_0}$  and  $(0.92 + c_2) M_{\delta_0}$  for Mach < 1 and Mach > 1, respectively. There are no plans to fly the F-8C with these modifications.

Several identification schemes were examined to perform these functions, including approximated (second-order) nonlinear filters, Lyapunov-stable model trackers, maximum likelihood estimation (MLE), and self-excited limit cycle systems. Trial designs were actually carried out for the latter three (ref. 3). The MLE procedure was selected out of these primarily because it offers the greatest capability in terms of the numbers of parameters identified and their accuracy. Details of the MLE design are discussed below.

*Identification accuracy-* Before proceeding with the identifier description, we present some evaluations of theoretical limits on the accuracy to which parameters of the F-8C can be identified under the ground rules imposed in section II. These evaluations provide justification for most of the design decisions made for the MLE identifier. The accuracy evaluations were performed for the identification problem illustrated in figure 7. The aircraft (as modeled in fig. 1) with a simple pitch rate feedback control loop was assumed to be excited by small test signals, wind gusts, and sensor noise. Discrete pitch rate, normal acceleration, and surface position observations were assumed available to identify unknown dynamic coefficients (as parameterized in table I), gust level, trim disturbances, and initial conditions. It is well known that the achievable identification accuracy for this problem is limited by the following lower bound (ref. 24):

$$\text{Cov}(\underline{c} - \hat{\underline{c}}) \geq M(\underline{c}, T) \triangleq E \left\{ \sum_{k=1}^{T/\Delta t} \frac{\partial v_k}{\partial \underline{c}} B_k^{-1} \left( \frac{\partial v_k}{\partial \underline{c}} \right)^T + P_0^{-1} \right\}^{-1} \quad (5)$$

where  $v_k$ ,  $k = 1, 2, \dots$ , is the sequence of residuals generated by a Kalman filter designed for the true parameter values;  $B_k$ ,  $k = 1, 2, \dots$ , is the covariance matrix of that sequence;  $P_0$  is the covariance matrix of initial parameter uncertainties; and  $T$  is the data length.

The situation in figure 7 was simulated, and sample averages of the matrix  $M$  were computed for various combinations of data length, test signal, gust, and sensor noise conditions. The principal result of these calculations is shown in table III, which compared square roots of selected diagonal elements of  $P_0$  with corresponding elements of  $M(\underline{c}, T)$ . The comparisons<sup>4</sup> are made for  $T = 5$  and  $T = 10$  sec and correspond to the following environmental conditions:

Flight condition: 20,000 ft, Mach = 0.67,  $\bar{q} \approx 305$  lb/ft<sup>2</sup>

Test signal: Square wave at short-period frequency,  
0.015 g RMS acceleration

Gust level: 1 ft/sec 1σ

Sensor noise:  $\sigma_g = 0.015^\circ/\text{sec}$ ,  $\sigma_{nz} = 0.02$  g

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<sup>4</sup>Because parameters can change significantly over 10 sec, it is not meaningful to look at longer data lengths.



Table III shows that we can expect only limited identification capability under these conditions. Our *a priori* knowledge improves for only two dynamic coefficients,  $M_{\delta_0}$  and  $c_2$  (the small perturbation of  $M_{\alpha}$ ), and for  $\sigma_{wg}$ , trim, and initial conditions (not shown). Accuracies are limited to approximately 10 percent for both  $M_{\delta_0}$  and  $M_{\alpha}$ . Fortunately, these are adequate in most gain-scheduling applications. Using similar analyses for other test conditions, the following additional conclusions were established:

**Accuracy limits:**  $M_{\delta_0}$  and  $M_{\alpha}$  errors are proportional to true parameter values. Accuracies fall into the same 10-percent range for all flight conditions.

**Reduced parameter identification:** Identifiers designed to find only a few parameters (as few as  $M_{\delta_0}$ , trim, and ICs) do almost as well as identifiers designed to find all parameters, even in the presence of errors in the unrecognized parameters. However, trim disturbances and initial angle of attack cannot be deleted without inducing major errors in  $M_{\delta_0}$ .

**Test signals:** Randomized test signals are superior to cyclic ones for reduced parameter identification.

**Lateral-directional identification:** Identifiers for the lateral-directional axes reduce *a priori* uncertainties for only two parameters,  $\bar{q}$  and  $N_{\beta}$  (yawing moment due to sideslip). Accuracies are comparable to longitudinal identification.

**MLE identifier structure-** In view of the above identifiability limitations for the F-8C aircraft,<sup>5</sup> it was decided to utilize a longitudinal-axis identifier only with a reduced parameter set of three components —  $M_{\delta_0}$ ,  $c_2$ , and  $c_3$  (small perturbation on velocity).<sup>6</sup> It was also decided to avoid the iterative nature of conventional MLE algorithms (refs. 9 and 10) by adopting a parallel approach to likelihood minimization. This approach is illustrated in figure 8. It consists of a bank of Kalman filters, each generating sufficient statistics for the conditional probability distribution of the measurements given a different value for the unknowns, that is,

$$p(y_1, y_2, \dots, y_{T/\Delta t} | c_i) = \prod_{k=1}^{T/\Delta t} N[\hat{y}_k(c_i), B_k(c_i)] \quad \left. \vphantom{\prod_{k=1}^{T/\Delta t}} \right\} \quad (6)^7$$

$$1 = 1, 2, \dots, N$$

<sup>5</sup>It is important to recognize that the limitations cited above are F-8C specific and are caused primarily by the low test signal ground rules imposed in section II.

<sup>6</sup>Though only weakly identifiable,  $c_3$  was carried along for flight experiments with larger test signals.

<sup>7</sup> $N[x, B]$  denotes the usual multivariate normal distribution with mean  $x$  and covariance matrix  $B$ .

where  $\hat{y}_k(\underline{c}_i)$  and  $B_k(\underline{c}_i)$  are predicted measurements and residual covariances, respectively, from the  $i$ th Kalman filter designed for  $\underline{c} = \underline{c}_i$ . Likelihood functions ( $L \triangleq -\ln p$ ) are then computed for each filter channel, the minimum likelihood channel is selected, and a single (approximate) Newton-Raphson step is taken from there to predict the parameter value, that is,

$$L(\underline{c}_i, T) = \sum_{k=0}^{T/\Delta t} \left[ \left\| y_k - \hat{y}_i(\underline{c}_i) \right\|_{B_k^{-1}(\underline{c}_i)}^2 + \ln \det B_k(\underline{c}_i) \right] \quad \left. \vphantom{\sum_{k=0}^{T/\Delta t}} \right\} \quad \begin{matrix} i = 1, 2, \dots, N \end{matrix} \quad (7)$$

$$i^* = \text{Arg} \left\{ \min_i L(\underline{c}_i, T) \right\} \quad (8)$$

$$\hat{\underline{c}} = \underline{c}_{i^*} - M(\underline{c}_{i^*}, T)^{-1} \frac{\partial L}{\partial \underline{c}}(\underline{c}_{i^*}, T) \quad (9)$$

where  $M(\underline{c}, T)$  is the approximate second-partials matrix defined in equation (5). This parallel minimization approach is a modified form of several procedures published in the literature (refs. 25-27). Its primary advantages for onboard use are recursiveness and fixed structure. The filters and sensitivity calculations implicit in equation (9) process measurement samples as they appear and use a fixed set of repetitive program instructions. No data storage and conditional iterative reprocessing are required. They also remain fixed in parameter space, requiring no onboard calculation of new plant models, Kalman filter gains, and their associated sensitivities. These are all desirable features for real-time onboard computer code.

*Design specifics-* A specific implementation of figure 8 involves the following design choices and issues:

- Identification models
- Channel selection
- Kalman filter design parameters
- Adjustments for practical situations not handled by the basic theoretical approach

These items are discussed below for the F-8C's specific design.

*Identification models-* Each Kalman filter channel in the F-8C design uses a discretized version of the basic aircraft model in figure 1 with the actuator state deleted ( $\delta_e$  is assumed measurable) and with the two trim disturbances appended as Brownian motion "trim states." These trim states eliminate the need (identified above) to treat the disturbances as explicit unknown parameters. The model's dynamic coefficients are parameterized as shown in table I, but  $c_1$ ,  $c_4$ , and  $c_5$  are assumed to be zero. Only  $c_2$ ,  $c_3$ , and  $M\delta_0$  are recognized as unknowns.

*Channel selection-* The three unknowns generate a parameter space illustrated in figure 9. Two axes are defined by  $M_{\delta 0}$  and  $c_2$  (or equivalently  $M_\alpha$ ) and the third axis (not shown) is defined by  $c_3$ . The problem of channel selection is to choose both the number and the location of points in this space at which to operate filters. We obviously want as few channels as possible, yet they must be close enough together to be able to interpolate with one Newton-Raphson step (eq. (9)). A bit of experimentation showed that five channels suffice for the F-8C (located as shown in the figure). Four cover the subsonic range and are logarithmically distributed along the  $M_{\delta 0}$  axis, with both  $c_2$  and  $c_3$  equal to zero. The logarithmic arrangement is motivated by our earlier finding that expected identification errors are proportional to true parameter values. The fifth channel covers supersonic flight regimes. Each channel interpolates over an approximate  $\pm 50$ -percent range about its nominal  $M_{\delta 0}$  value.

*Kalman filter design-* Kalman filters and sensitivity filters for each channel were designed under steady-state assumptions, using the following fixed values for gust intensity, sensor noise, and trim disturbance growth rates:

$$\begin{aligned} \sigma_{\omega g} &= 6 \text{ ft/sec} \\ \sigma_g &= 0.15^\circ/\text{sec} \\ \sigma_{nz} &= 0.02 \text{ g} \\ \left. \begin{aligned} \sigma_{g/V} &= 0.017 \text{ (g/V)}_i / \sqrt{\text{sec}} \\ \sigma_{M_0} &= 0.001 \text{ (M}_{\delta 0})_i / \sqrt{\text{sec}} \end{aligned} \right\} \quad i = 1, 2, \dots, 5 \end{aligned}$$

In addition, the surface position measurement was assumed to be corrupted by discrete additive white noise with  $0.04^\circ$  rms intensity. These numbers can be considered roughly representative of the F-8C's environment.

*Adjustments-* The basic MLE design required surprisingly few after-the-fact "fixes" to make it work in situations not wholly consistent with the theoretical maximum likelihood problem formulation. Two such adjustments are worth mentioning here. The first deals with the fact that the aircraft's parameters are not really constant, as assumed in the design so far. This means that we cannot accumulate likelihood functions and their sensitivities indefinitely, as implied by equations (5) to (9). Rather we must "forget" past data by some expedient such as the high-pass operations used in figure 8. The high-pass approach was selected primarily because it preserves the overall recursive nature of the algorithm. Time constants ( $\tau$ ) were set at 5 sec, an experimental compromise between systematic tracking errors occurring for  $\tau$  too large and random errors or false minimums occurring for  $\tau$  too small.

The second adjustment deals with time-varying noise statistics. These can cause substantial estimation errors in identifiers designed for fixed statistics. Consider, for example, a case where gust, sensor noise, and trim disturbances are actually very small compared to the nominal design values.

Then the likelihood function in equation (7) will be dominated by the  $\ln \det B(\underline{c}_i)$  terms. These depend on *a priori* channel data only, not on incoming measurements. For the F-8C, they are minimized at channel 1,  $M_{\delta 0} = -2.34$ , regardless of the true  $M_{\delta 0}$  value. Hence they can produce large estimation errors. This potential problem was alleviated by scaling the  $\ln \det B$  terms (and their gradients) with the constant

$$\sigma^2 = \min_i \sum_{k=0}^{T/\Delta t} \|y_k - \hat{y}_k(\underline{c}_i)\|_{B_k^{-1}(\underline{c}_i)}^2 / (2T/\Delta t) \quad (10)^8$$

which can be formally derived by considering  $\sigma$  to be an unknown common scale factor on all noise statistics to be identified as part of the MLE procedure (ref. 3).

The F-8C identifier incorporates several other less important adjustments not covered here. These include interpolation of state estimates between channels, automatic initialization and data transfer when channel changes occur, significance tests on channel selections, and high-frequency roll-offs on sensitivity accumulation to prevent sharp estimation transients. These are covered in reference 3. Reference 3 also provides computer sizing estimates for the complete identifier. These call for approximately 2200 words of memory and 6 msec of execution time per data sample.

#### SECTION IV - ADAPTIVE SYSTEM PERFORMANCE

The control laws and identification algorithms described in the last section are scheduled for flight evaluation during 1976-77. In preparation for these tests, they have been mechanized and evaluated on the F-8 simulator at Langley Research Center and are also scheduled for evaluation on the simulator at Dryden Flight Research Center. Only Langley's evaluations are available to date. Key results are described below.

Langley's F-8 simulator is an extensive six-degree-of-freedom nonlinear digital simulation program. It includes complete aircraft dynamics, actuators, servos, flexibility, sensor models, and real-time interfaces with an F-8 iron-bird cockpit and with standard simulation consoles (ref. 14). This facility was used to evaluate four "measures of goodness" of the adaptive system:

- Identification accuracy in steady flight
- Convergence characteristics
- Tracking characteristics for standard flight transition

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<sup>8</sup>As in equations (5) to (9), the accumulation in equation (10) and the growing divisor  $(2T/\Delta t)$  are actually high-passed.

- Responses to major maneuvers and configuration changes

These criteria deliberately concentrate on identifier performance. Overall closed-loop performance, as mentioned earlier, provides a less sensitive measure of adaptive capability.

#### Accuracy at Fixed Flight Condition

Identification accuracy (and thus control gain accuracy) was verified by subjecting the closed-loop adaptive system to a standard sequence of test conditions while in trimmed flight at several fixed flight conditions. The test sequence consisted of a period of quiet, followed by a 20-ft/sec<sup>2</sup> pilot doublet C\* command, followed by a period of atmospheric turbulence, and then again by a 20-ft/sec<sup>2</sup> doublet. This entire sequence was repeated with and without sensor noise and always includes a small random C\* command as a test signal. Accuracy results for these tests are summarized in table IV and verify the theoretically predicted identifier performance. Typical transient responses are given in figure 10.

#### Convergence Characteristics

To verify convergence properties, the algorithm's min-select was initialized at a preselected channel and required to converge to a flight condition covered by another channel. This was done for several combinations of channels and input conditions under normal closed-loop operation. Convergence times (to within  $\pm 20$  percent of final  $M_{\delta 0}$  value) were less than 1 sec for all cases. Typical examples are shown in figure 11.

#### Tracking

The system's ability to track changing aircraft parameters was tested by flying through standard flight transitions. One such transition is shown in figure 12. It consists of maximum level accelerating flight starting at low dynamic pressure,  $h = 20,000$  ft, Mach 0.40,  $\bar{q} = 109$  psf, and terminating just above Mach 1.0. To achieve repeatability, the transition was flown without pilot inputs. We simply applied 100-percent military power plus full afterburner and held pilot-commanded C\* and lateral-directional commands at zero. The resulting transients in figure 12 verify smooth tracking behavior and proper channel changes as dynamic pressure builds up. Note, in particular, that  $\hat{M}_\alpha$  is discontinuous as the identifier decides that the aircraft has gone supersonic. This jump is consistent with the parameterization in table I. Figure 12 includes sensor noise and 6-ft/sec RMS vertical gusts.

#### Major Maneuvers

The closed-loop adaptive system was also tested under large maneuver conditions and configuration changes. These included various rolling maneuvers, high-g turns, step angle-of-attack and sideslip disturbances, and speed

brake-, gear-, and wing-transition transients. These were largely flown under pilot control from the F-8 iron-bird cockpit. They were intended to explore qualitative properties of the concept under large signal condition and to assure that no drastic upsets occur during normal flight operations. Transient traces are documented in reference 3.

## SECTION V - CONCLUSIONS AND REMAINING ISSUES

The nonlinear simulation evaluations in section IV verify that the parallel channel maximum-likelihood identifier works well throughout the flight envelope and provides adequate gain-scheduling signals for the control law. This is ample motivation to proceed to flight test. Like most system designs, however, the complete adaptive controller is not ideal and it is important to highlight remaining issues and limitations. These include tracking errors, compromises due to fixed filter design statistics, and test signal requirements.

The tracking error limitation is evident in figure 12. Because of our 5-sec (effective) likelihood accumulation time, parameter estimates tend to lag in the presence of rapid parameter changes. These lags are responsible for the largest errors seen in the simulation trials. Errors as high as 30 percent are evident in the  $\Delta \bar{q}$  trace in figure 12, and 50 to 70 percent errors have been observed during configuration changes and more dramatic flight transitions (e.g., high-g pulldowns). Research is presently under way to provide some means of dealing with these time variations.

The second limitation is evident in most of the accuracy runs. Parameter estimates shift somewhat between quiet or sensor-noise-only conditions and turbulence conditions. This appears to be caused by inconsistencies between true environmental statistics and the statistics used to design Kalman filter channels. Some on-line adjustment of these statistics may be desirable.

Test signal requirements pose a potential limitation not quantifiable at this point. Required levels are determined primarily by small nonlinearities and actual noise characteristics in the flight-control hardware, while acceptable levels are determined by pilot considerations. In the simulations at Langley Research Center, a small random  $C^*$  command signal (white noise passed through the second-order filter  $s/(s^2 + 15s + 36)$ ) was found adequate. Its RMS magnitude was adjusted to  $C^* = 4 \text{ ft/sec}^2$ , which produced the following approximate accelerations at the pilot station:

<u>Flight condition</u>	<u>Test signal acceleration</u>
Low $\bar{q}$ ( $h = 20,000$ , $M = 0.4$ )	0.01 g RMS
Medium $\bar{q}$ ( $h = 20,000$ , $M = 0.67$ )	.025 g RMS
High $\bar{q}$ ( $h = 3000$ , $M = 0.8$ )	.04 g RMS
Supersonic ( $h = 40,000$ , $M = 1.2$ )	.02 g RMS

These levels can be lowered somewhat with tolerable accuracy degradations. They cannot be used with much certainty, however, to predict the levels

needed for the actual aircraft. For these requirements, we must wait until flight.

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TABLE I.- F-8 LONGITUDINAL MODEL PARAMETERIZATION

Equation coefficient		Small perturbation parameter	Perturbation parameter uncertainty
$M_q = -0.23 + 0.028 M_{\delta o} + c_1$	Mach < 1	$c_1$	0.065
$= -0.23 + 0.010 M_{\delta o} + c_1$	Mach > 1		
$M_{\alpha} = (0.61 + c_2)M_{\delta o}$	Mach < 1	$c_2$	0.135
$= (1.53 + c_2)M_{\delta o}$	Mach > 1		
$V = (200 + c_3) - M_{\delta o}$	Mach < 1	$c_3$	31.5
$= (260 + c_3) - M_{\delta o}$	Mach > 1		
$Z_{\alpha}V = (53 + c_4)M_{\delta o}$	All Mach	$c_4$	5.0
$^a M_{\delta o} = c_5$	All Mach	$c_5$	15.0
$Z_{\delta}V = (7.7 + c_6)$	All Mach	$c_6$	0.75
<hr/>		<hr/>	
$\bar{q} = (2 + c_7)M_{\delta o}$	All Mach	$c_7$	2.0

<sup>a</sup>This is "unflexed" surface effectiveness. Actual surface effectiveness taking quasi-static structural bending into account is given by

$$M_{\delta} = (1 + 0.016M_{\delta o} + 0.0002M_{\delta o}^2)M_{\delta o}.$$

TABLE II.- LONGITUDINAL WEIGHTS AND GAINS

Flight condition	20,000	20,000	40,000	10,000
Altitude (h) ft				
Mach M	0.67	0.40	1.2	0.8
Dynamic pressure ( $\bar{q}$ ) psf	305	109	395	652
Quadratic weights				
$q_1$ on $\dot{x}_3$	100	1,000	1,000	100
$q_2$ on $\dot{x}_3 = C^* - C^*_c$	10	100	100	1
$q_3$ on $\delta_e$	1,000	1,000	10,000	1,000
$q_4$ on $\delta_c$	1,000	1,000	1,000	1,000
Selected control gains				
$q$ to $\delta_c$	0.30	0.96	0.19	0.24
$n_z$ to $\delta_c$	.0086	.0011	.0017	-.000012
$x_3$ to $\delta_c$	.040	.13	.040	.013
$\delta_e$ to $\delta_c$	.23	.11	.13	.056

TABLE III.- IDENTIFICATION ACCURACY LIMITS FOR THE F-8C

Parameter (from table I)	True value	Initial uncertainty	1- $\sigma$ uncertainty at T = 5	1- $\sigma$ uncertainty at T = 10
$c_1$	0	0.065	0.063	0.062
$c_2$	0	.013	.065	.056
$c_3$	0	31.5	30.5	29.7
$c_4$	0	5.0	4.6	4.4
$M_{\delta_0}$	-13.8	15.0	1.46	1.13
$c_5$	0	.75	.75	.75
<hr/>				
$\sigma_{wg}$	1.0	1.5	.35	.25
$M_0$	0	.125	.0063	.0050

TABLE IV.- F-8C IDENTIFIER ACCURACY AT  
FIXED FLIGHT CONDITIONS

Conditions	Percent error <sup>a</sup>				
	$\hat{M}_{\delta o}$	$\hat{M}_{\alpha}$	$\hat{V}$	$\hat{q}$	$\hat{\alpha}$
<b>h = 20,000, M = 0.67</b>					
Quiet	≤4	10	4	2	5
Gusts & sensor noise	7	≤8	≤3	10	
Pilot commands	≤4	≤8	6	2	
<b>h = 20,000, M = 0.4</b>					
Quiet	≤11	≤15	8	2	8
Gusts & sensor noise	≤11	≤15	≤5	10	
Pilot commands	≤11	≤15	8	4	
<b>h = 40,000, M = 1.2</b>					
Quiet	≤3	2	2	16	≤16
Gusts & sensor noise	12	2	4	30	
Pilot commands	≤3	2	2	15	
<b>h = 3,000, M = 0.8</b>					
Quiet	2	17	40	2	≤9
Gusts & sensor noise	23	9	23	15	
Pilot commands	6	9	29	4	

<sup>a</sup>Errors were computed by comparing linearized coefficients with estimates plotted in real time on strip charts. Many errors fall below the expected resolution of this process.

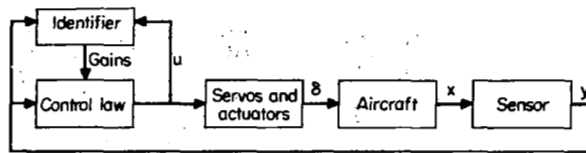


Figure 1.- Adaptive control law structure.

State Dynamics:

$$\frac{d}{dt} \begin{bmatrix} q \\ \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} M_q & M_\alpha & M_\delta \\ 1 & Z_\alpha & Z_\delta \\ 0 & 0 & -a \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix} \delta_c + \begin{bmatrix} M_0 \\ g/V \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/\sqrt{875V} \sigma_{wg} \quad n_{wg} \\ 0 \end{bmatrix}$$

Measurements (discrete):

$$y_k = \begin{bmatrix} q \\ \alpha \\ n_{zk} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ dM_q & dM_\alpha - Z_\alpha V & dM_\delta - Z_\delta V \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ \delta_{ek} \end{bmatrix} + \begin{bmatrix} \sigma_g \quad n_{gk} \\ \sigma_{nz} \quad n_{nz_k} \end{bmatrix} \quad k = 0, 1, 2, \dots$$

Definitions:

$q$	pitch rate	$a$	actuator bandwidth (12 r/sec)
$\alpha$	total angle of attack	$d$	accelerometer displacement from C.G. (15 ft)
$\delta_e$	elevator position	$\sigma_{wg}$	vertical gust level (ft/sec rms)
$\delta_c$	elevator command	$\sigma_g, \sigma_{nz}$	sensor noise levels
$n_z$	normal acceleration	$M_q, M_\alpha, M_\delta,$	dynamic coefficients (functions of $c$ )
$n_{wg}$	unit intensity white noise	$Z_\alpha V, Z_\delta V, V$	
$n_{gk}, n_{nz_k}$	unit variance white sequences	$M_0, g/V$	constant trim disturbances (functions of $c$ )

Figure 2.- F-8C longitudinal design model.

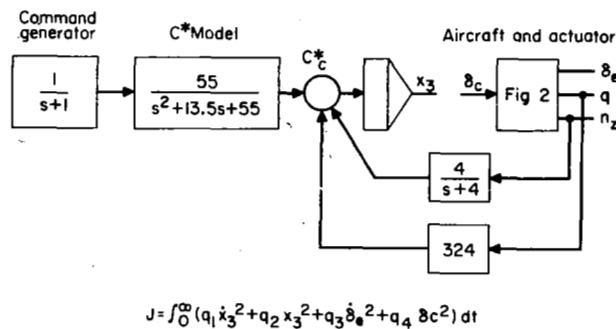


Figure 3.- Longitudinal optimization problem.

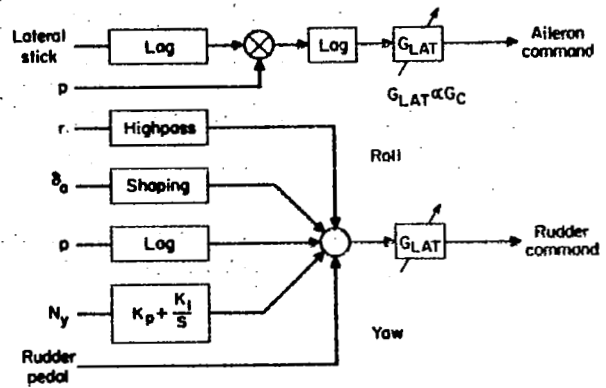
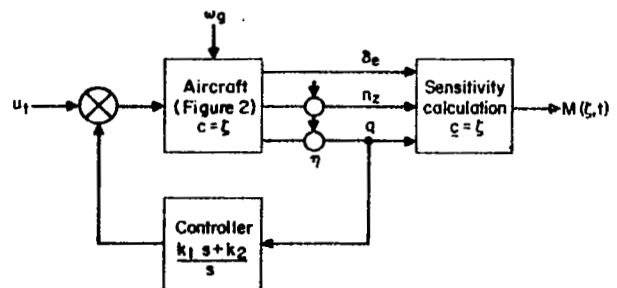
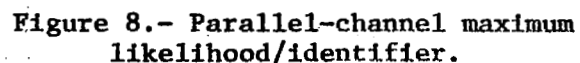


Figure 6.- Lateral-directional control law.



**Figure 7.- Identification accuracy analysis.**



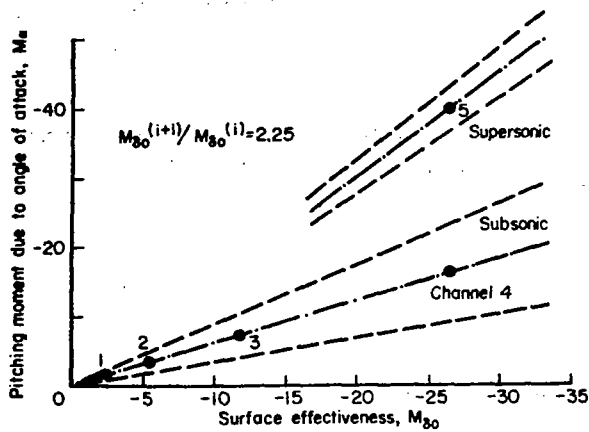


Figure 9.- F-8C identifier channel locations.

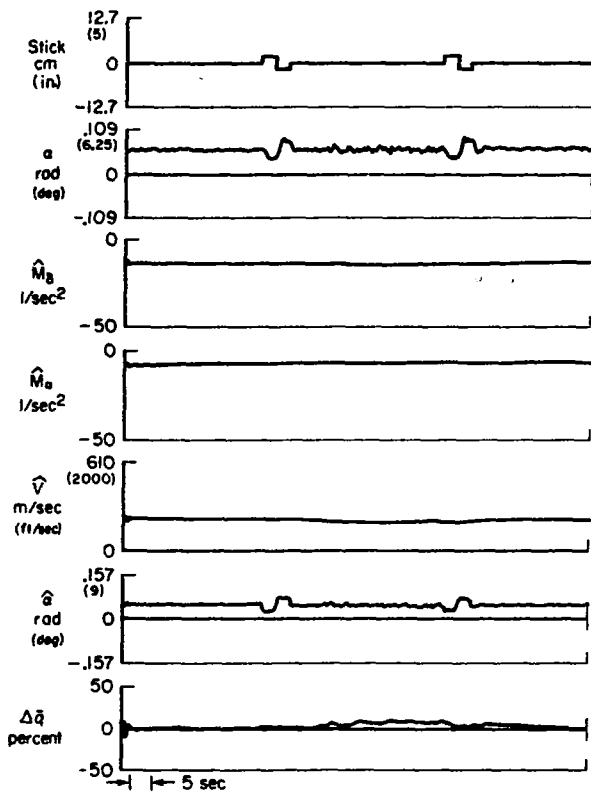


Figure 10.- Typical accuracy test traces;  
 $h = 20,000$ ,  $M = 0.67$ .

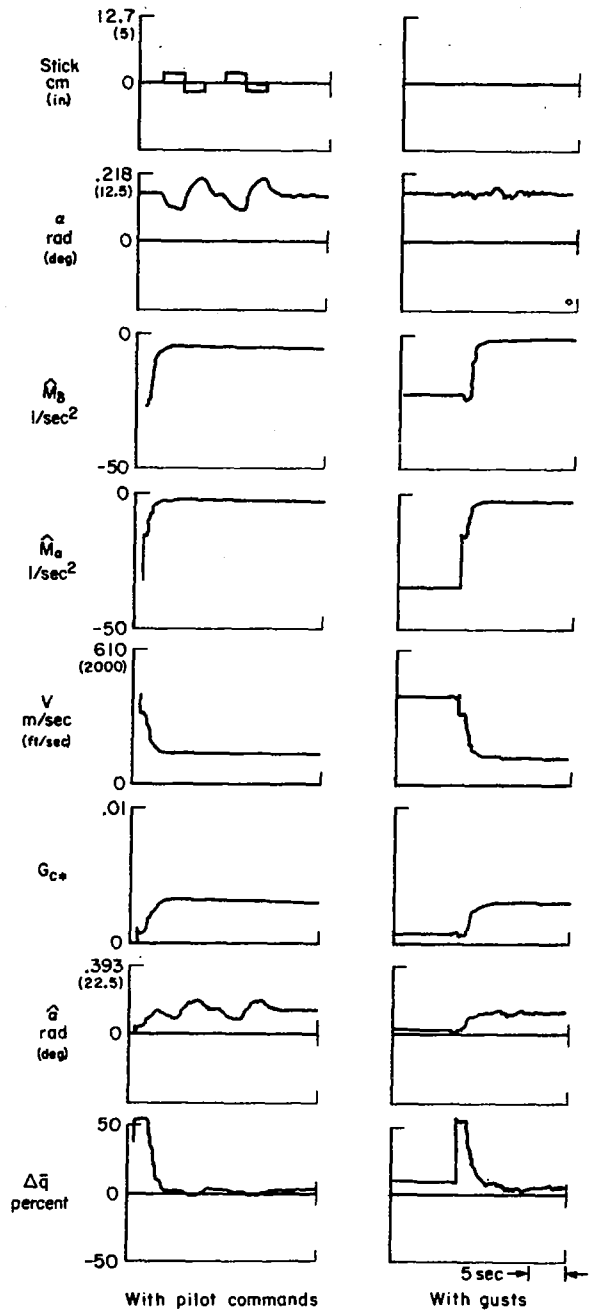


Figure 11.- Typical convergence transients;  
 $h = 20,000$ ,  
 $M = 0.4$ .

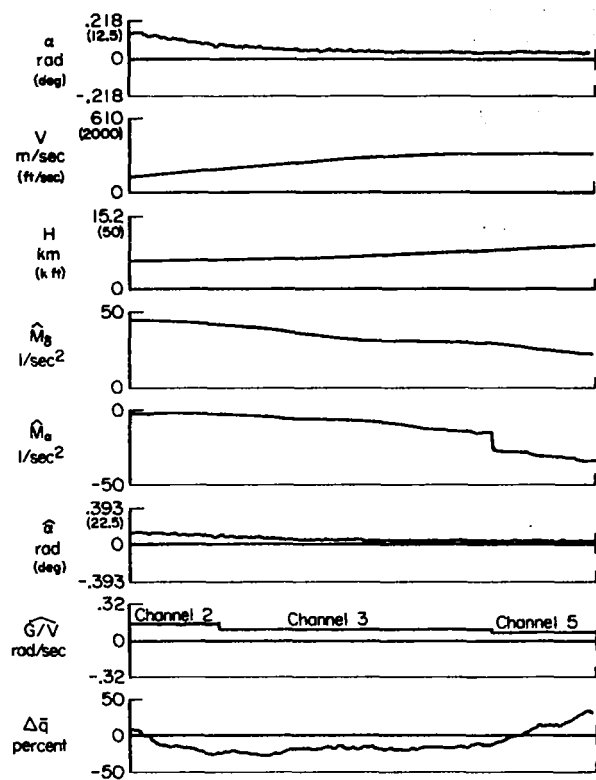


Figure 12.- Tracking traces.

1. The first part of the document is a list of the names of the persons who have been appointed to the various offices of the city.

2. The second part of the document is a list of the names of the persons who have been appointed to the various offices of the city.

3. The third part of the document is a list of the names of the persons who have been appointed to the various offices of the city.

4. The fourth part of the document is a list of the names of the persons who have been appointed to the various offices of the city.

5. The fifth part of the document is a list of the names of the persons who have been appointed to the various offices of the city.

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12. The twelfth part of the document is a list of the names of the persons who have been appointed to the various offices of the city.



# SELF-TUNING REGULATORS

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## 1. INTRODUCTION

This paper gives a brief review of the results of a research project on self-tuning regulators which has been carried out at the Lund Institute of Technology. The project is part of a larger research program on adaptive control which has followed three main lines: A - stochastic control, B - self-tuning regulators, and C - analysis of adaptive regulators proposed in literature.

The approach via stochastic control, which leads to dual-control strategies, has been very useful to provide understanding and insight. So far the results have, however, not been carried out to the stage of implementation. Self-tuning regulators are a particular version of adaptive regulators based on real-time identification. They are a special case of nondual stochastic control algorithms. The work on self-tuning regulators has progressed quite far in the sense that these regulators are reasonably well understood theoretically. They have also been tried extensively in several industrial applications. On the other hand, much work remains to be done in exploring other aspects of these regulators. Project C is needed to stay abreast of the development of other adaptive schemes. It has also resulted in a long list of problems relating to understanding the strange behavior of some algorithms in certain circumstances.

The basic idea underlying the self-tuning regulators is the following. If a description of a system and its environment is known, there are many procedures available to design a control system subject to given specifications. When trying to remove the assumption that the models for the system and its environment are known, we are immediately led to the problem of controlling a system with constant but unknown parameters. This problem can, in principle, be solved by using stochastic control theory at the price of exorbitant calculations. It is then meaningful to ask if there are simple control algorithms that do not require information about the model parameters, such that the controller will converge to the controllers that could be designed if the model parameters were known. It is an empirical fact that such controllers exist in several cases. The investigation of their properties has also led to powerful tools that can be used to analyze many other cases.

The generation of self-tuning algorithms is partly heuristic. It turns out that many algorithms can be obtained by combining a real-time identifier with a control scheme. In our work we have so far mostly considered regulators for the LQG regulator problem. This has been motivated by the particular

applications we have considered. Many of the concepts and ideas can, however, be extended to many other design methods.

## 2. AN EXAMPLE

The main ideas are first demonstrated by a simple example. Consider the simple discrete time system:

$$y(t+1) + ay(t) = bu(t) + e(t+1) + ce(t) \quad (1)$$

where  $u$  is the input,  $y$  the output, and  $\{e(t)\}$  a sequence of independent, equally distributed, random variables. The number  $c$  is assumed to be less than 1. Let the criterion be to minimize the variance of the output, that is,

$$\min V = \min E y^2 = \min E \frac{1}{t} \sum_{k=1}^t y^2(k) \quad (2)$$

It is easy to show that the control law

$$u(t) = \frac{a-c}{b} y(t) \quad (3)$$

is a minimum variance strategy, and that the output of system (1) with feedback (3) becomes

$$y(t) = e(t) \quad (4)$$

(see, e.g., Åström (ref. 1)). Note that the control law (3), which represents a proportional regulator, can be characterized by one parameter only.

A self-tuning regulator for the system (1) can be described as follows:

ALGORITHM (Self-Tuning REGulator)

Step 1 (Parameter Estimation)

At each time  $t$ , fit the parameter  $\alpha$  in the model

$$\hat{y}(k+1) + \alpha y(k) = u(k), \quad k = 1, \dots, t-1 \quad (5)$$

by least squares, that is, such that the criterion

$$\sum_{k=1}^t \epsilon^2(k) \quad (6)$$

where

$$\epsilon(k) = y(k) - \hat{y}(k) \quad (7)$$

is minimal. The estimate obtained is denoted  $\alpha_t$  to indicate that it is a function of time.

### Step 2 (Control)

At each time  $t$ , choose the control

$$u(t) = \alpha_t y(t) \quad (8)$$

where  $\alpha_t$  is the estimate obtained in step 1.

### Motivation

There are several ways to arrive at the control strategy given above. The algorithm STURE can, for example, be interpreted as the *certainty equivalence* control for the corresponding stochastic control problem.

### Analysis

The properties of a closed-loop system controlled by a self-tuning regulator are now discussed. Since the closed-loop system is nonlinear, time-varying, and stochastic, the analysis is not trivial.

It is fairly obvious that the regulator will perform well if it is applied to a system (1) with  $b = 1$  and  $c = 0$ , because in this case the least-squares estimate  $\alpha_t$  will be an unbiased estimate of  $a$ . The regulator (8) will thus converge to a minimum variance regulator if the parameter estimate  $\alpha_t$  converges. It is surprising, however, that the regulator will also converge to the minimum variance regulator if  $c \neq 0$  (as demonstrated below). There may also be some difficulties because the control law is of the certainty equivalence type. Because of the special model structure (5), the feedback gain will, however, be bounded if the estimate  $\alpha_t$  is bounded.

The least-squares estimate is given by the normal equation

$$\frac{1}{t} \sum_{k=1}^t y(k+1)y(k) + \alpha_{t+1} \frac{1}{t} \sum_{k=1}^t y^2(k) = \frac{1}{t} \sum_{k=1}^t y(k)u(k)$$

Assuming that the estimate  $\alpha_t$  converges toward a value that gives a stable closed-loop system, then it is straightforward to show that

$$\frac{1}{t} \sum_{k=1}^t (\alpha_{t+1} - \alpha_k) y^2(k) \rightarrow 0$$

Thus the closed-loop system has the property

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{k=1}^t y(k+1)y(k) = 0 \quad (9)$$

Furthermore, assuming that the system to be controlled is governed by equation (1), the output of the closed-loop system obtained in the limit is given by

$$y(t) + [a - \alpha b]y(t-1) = e(t) + ce(t-1) \quad (10)$$

The covariance of  $\{y(t)\}$  at lag 1 is then given by

$$E y(t+1)y(t) = -f(\alpha) = \frac{(c - a + \alpha b)(1 - ac + \alpha bc)}{1 - (a - \alpha b)^2} \quad (11)$$

Condition (9) gives

$$f(\alpha) = 0$$

A second-order equation for  $\alpha$  which has the solutions:

$$\alpha = \alpha_1 = \frac{a - c}{b}$$

$$\alpha = \alpha_2 = \frac{a - (1/c)}{b}$$

The corresponding poles of the closed-loop system are  $\lambda_1 = c$  and  $\lambda_2 = 1/c$ , respectively. Since  $c$  was assumed less than 1, only the value  $\alpha_1$  corresponds to a stable closed-loop system. Note that  $\alpha_1$  corresponds to the gain of the minimum variance regulator (3). Hence, if the parameter estimate  $\alpha_t$  converges to a value that gives a stable closed-loop system, then the closed-loop system obtained must be such that equation (9) holds. This means that the algorithm can be thought of as a regulator that attempts to bring the covariance of the output at lag 1, that is,  $r_y(1)$ , to zero in the same way as an integrating regulator brings the integral of the control error to zero.

If the system to be controlled is actually governed by equation (1), then the self-tuning regulator will converge to a minimum variance regulator if it converges at all.

Figure 1 shows the results of a simulation of the algorithm. It is clear from this simulation that the algorithm converges in the particular case. The least-squares estimate will be a biased estimate of the model parameter  $a = -0.5$  because of the correlation between the model errors. As can be expected from the previous analysis, the bias is, however, such that the limiting regulator corresponds to the minimum variance regulator. The lower part of figure 1 shows the asymptotic value of the loss function obtained if the regulator gain is fixed to the current value. It is clear from this figure that the loss function is very close to the minimum loss for the case of known parameter after 50 steps.

### 3. GENERALIZATIONS

A regulator that generalizes the simple self-tuner of the previous section is shown in figure 2. The regulator can be thought of as being composed of three parts: a parameter estimator (block 1), a controller (block 3), and a third part (block 2), which relates the controller parameters to the estimated parameters. The parameter estimator acts on the process inputs and outputs and produces estimates of certain process parameters. The controller is simply a linear filter characterized by the coefficients of its transfer function. These coefficients are generally a nonlinear function of the estimated parameters. This function is frequently not one to one. This way of describing the regulator is convenient from the point of view of explaining how it works. The subdivision is, however, largely arbitrary, and the regulator can equally well be regarded simply as one nonlinear regulator. The functions of blocks 1, 2, and 3 are also simple, but the interconnection of these blocks represents a system with a rather complex input-output relation. The partitioning of the regulator (fig. 2) is also convenient from the point of view of implementation because the parameter estimator and the controller parameter calculation are often conveniently time shared between several loops.

There are many different ways to estimate the parameters  $\theta$  and to calculate the regulator parameters,  $\vartheta$ . Some possibilities are shown in figure 3. The complexity of the algebraic equation that relates the controller parameters to the estimated parameters can vary significantly, from a simple variable substitution for minimum variance regulators to solution of an algebraic Riccati equation for the general LQG case.

#### Analysis

A brief statement of some properties of the self-tuning regulators are now given. The results are fairly technical and only a few main points are given here. A review of available results are given in reference 2. The major results were proven in references 3 to 5.

For the analysis, it is assumed that the process to be controlled is governed by

$$A(q^{-1})y(t) = B(q^{-1})u(t - k) + v(t) \quad (12)$$

where  $A(q^{-1})$  and  $B(q^{-1})$  are polynomials in the backward shift operator  $q^{-1}$  and  $\{v(t)\}$  is a sequence of random variables with bounded fourth moment. The analysis will basically cover the case  $\mu(t) \rightarrow 0$  as  $t \rightarrow \infty$ , which corresponds to the case when the parameters are constant.

The following problems can be resolved partially by analysis:

- Overall stability of the closed-loop system
- Convergence of the regulator
- Properties of the possible limiting regulators

The analysis is far from trivial because the closed-loop system is a nonlinear, time-variable stochastic system. Even if the recursive identification schemes used are well known, their convergence properties are largely unknown except for the least-squares case. The input is also generated by a time-varying feedback, which introduces additional difficulties. If the process noise  $\{v(t)\}$  is correlated, the least-squares estimates will be biased and the bias will depend on the feedback used.

A global stability result was proven by Ljung and Wittenmark (refs. 5 and 6) (see fig. 4). This result applies to a regulator composed of a least-squares identifier and a minimum variance controller. The result requires that the system (12) is minimum phase and that the time delay  $k$  and the parameter  $\beta_0$  are known.

A key result in the analysis is the observation made by Ljung (ref. 4) that the paths of the estimates are closely related to the trajectories of the differential equation:

$$\left. \begin{aligned} \frac{d\theta}{d\tau} &= Sf(\theta) \\ \frac{dS^{-1}}{d\tau} &= G(\theta) - S^{-1} \end{aligned} \right\} \quad (13)$$

where

$$\left. \begin{aligned} f(\theta) &= E[\Psi^T(t, \theta) \varepsilon(t, \theta)] \\ G(\theta) &= E[\Psi^T(t, \theta) \Psi(t, \theta)] \end{aligned} \right\} \quad (14)$$

In the particular case of the regulator LS+MV, the control law is chosen in such a way that  $\hat{y}(t, \theta) = 0$  and the stationary points are then given by

$$\theta = f(\theta) = E[y(t+1)\phi(t)] = 0$$

The regulator LS+MV thus attempts to zero the autocovariance of the output and the crosscovariance of the input and the output for certain lags. This result, which generalizes the simple example discussed in section 2, was shown in reference 7. It was also shown here that

$$f(\theta) = 0 \quad (15)$$

has only one stationary solution for the regulator of LS+MV if the orders of the system and the model are compatible.

The differential equations (13) and (14) can be used in several different ways. Ljung has exploited them to construct both convergence proofs and examples which show that the parameter estimates do not converge. The differential equations have also been very useful in simulations (see, e.g., refs. 8 and 9).

The simulations shown in figures 5, 6, and 7 illustrate the behavior of different versions of the self-tuner.

#### 4. SERVOPROBLEM

So far, the self-tuning regulator has been discussed only in the framework of the regulator problem. It is straightforward to apply self-tuning to the servoproblem, too. Clarke and Gawthrop (ref. 10) propose to do so by posing a linear quadratic problem for a servoproblem.

Another approach is simply to introduce the reference values by the standard procedure using feedforward and an inverse model. For known parameters, the problem is handled as follows. Assume that the process is described by equation (12) and introduce the reference values  $u^r(t)$  and  $y^r(t)$  which satisfy the same dynamics as the process

$$A(q^{-1})y^r(t) = B(q^{-1})u^r(t - k) \quad (16)$$

Hence

$$A(q^{-1})[y(t) - y^r(t)] = B(q^{-1})[u(t - k) - u^r(t - k)] + v(t)$$

A design procedure for the regulator then gives the feedback

$$u(t) - u^r(t) = \frac{G(q^{-1})}{F(q^{-1})} [y(t) - y^r(t)]$$

If the command signal  $y^r(t)$  is specified,

$$u(t) = \frac{A(q^{-1})}{B(q^{-1})} y^r(t + k) + \frac{G(q^{-1})}{F(q^{-1})} [y(t) - y^r(t)] \quad (17)$$

This system cannot be realized unless the change in reference value is known or can be predicted  $k$  steps ahead. If this is not the case, a time delay in the response of  $k$  units must be accepted.

Observe that the control law (17) can be written:

$$B(q^{-1})F(q^{-1})u(t) = B(q^{-1})G(q^{-1})y(t) + [A(q^{-1})F(q^{-1}) - q^{-k}G(q^{-1})]y^r(t+k) \quad (18)$$

The servoproblem can be incorporated into the self-tuning regulator simply by changing the model in the parameter estimation step to

$$M: \hat{y}(t) = -A(q^{-1})y(t-1) + B(q^{-1})u(t-k) + C(q^{-1})\epsilon(t-1) \\ + D(q^{-1})y^r(t-1)$$

and making the modification (18) in the control step.

## 5. APPLICATIONS

The self-tuning regulators are conveniently implemented using a digital computer. The simple regulator LS+MV requires no more than 30 lines of FORTRAN code, while the regulator RML+LQ requires an order-of-magnitude more code because of the necessity of solving the algebraic Riccati equation in each iteration. The regulators have been applied to a number of industrial processes. Among the applications currently known to me are

- paper machine (refs. 11 and 12)
- digester (ref. 13)
- ore crusher (ref. 14)
- enthalpy exchanger (ref. 15)
- supertanker (ref. 16)

Several of these applications have been in operation for a long time. A self-tuning regulator has, for example, been running as an adaptive autopilot for a supertanker for more than a year.

Even if the regulators discussed automatically tune their parameters, it is necessary to determine some parameters in advance; for instance,

- Number of parameters in the prediction model ( $p$ ,  $r$ , and  $s$ )
- Initial values of the parameter estimates
- Value of any fixed parameters in the model
- Rate of exponential forgetting of past data in the estimation algorithm
- Sampling rate

Experience has shown that it is fairly easy to make the proper choice in practice. These parameters are also much easier to choose than to directly



determine the coefficients of a complex control law. It is our experience that system engineers without previous exposure to this type of algorithm have been able to learn how to use it after only a short training period. There have also been several misapplications. The most common mistake is to attempt a self-tuner for a control design that will not work even if the parameters are known.

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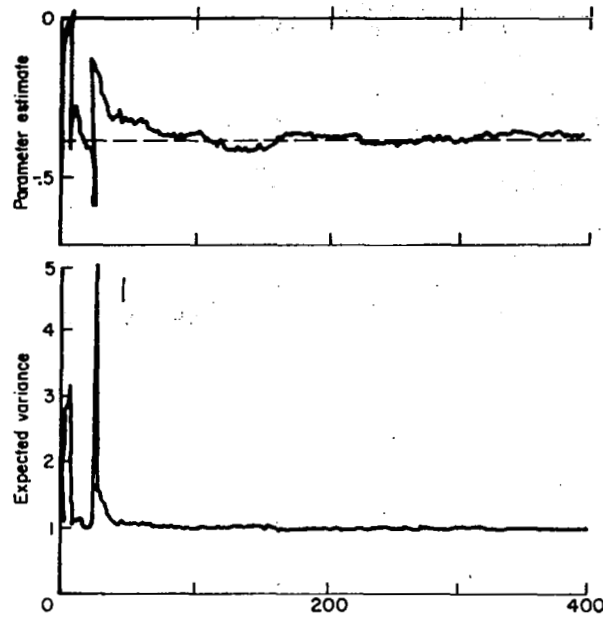


Figure 1.- Example of self-tuning regulator.

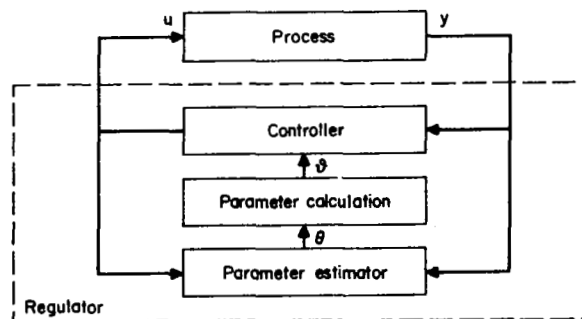


Figure 2.- Block diagram of a general self-tuning regulator.

### PARAMETER ESTIMATION

$$\begin{aligned}\text{Model: } \hat{y}(t) &= -A(q^{-1})y(t-1) + B(q^{-1})u(t-1) + C(q^{-1})\varepsilon(t-1) \\ &= \vartheta(t)\theta\end{aligned}$$

$$\varepsilon(t, \theta) = y(t) - \vartheta(t)\theta$$

$$\theta(t+1) = \theta(t) + \mu(t)S(t+1)\psi^T(t)\varepsilon(t, \theta)$$

$$S^{-1}(t+1) = S^{-1}(t) + \mu(t+1)[\psi^T(t+1)\psi(t+1) - S^{-1}(t)]$$

For: Least squares,  $C \equiv 0$

Extended least squares,  $\phi(t) = \psi(t)$

Recursive maximum likelihood,  $-\text{grad}_{\theta}\varepsilon(t, \theta)$

### CONTROL STRATEGIES

$$\mu(t) = \frac{G(q^{-1})}{F(q^{-1})} y(t)$$

Regulator parameters:  $\vartheta = \text{col}[q_1 q_2 \dots q_m ; f_1, f_2 \dots f_2]$

Criteria: Minimum variance

Linear quadratic

Figure 3.- Some approaches to parameter estimation and control.

Let the system be

$$A(q^{-1})y(t) = B(q^{-1})u(t - k) + V(t)$$

where the parameters are estimated by least squares and control gives a minimum variance response if the time delay,  $k$ , and the lead coefficient of

the polynomial  $B(q^{-1})$ ,  $\beta_0$ , are known if the system order is not underestimated and if

$$\limsup \frac{1}{N} \sum V^2(t) \leq \infty$$

then

$$\limsup \frac{1}{N} \sum y^2(t) < \infty$$

and if the system be minimum phase then also

$$\limsup \frac{1}{N} \sum u^2(t) < \infty$$

Figure 4.- An example global stability result.



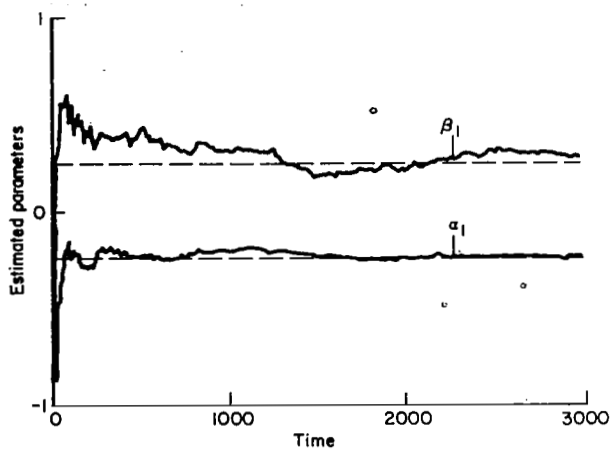


Figure 5.- Example regulator design using least squares and minimum variance estimation and control.

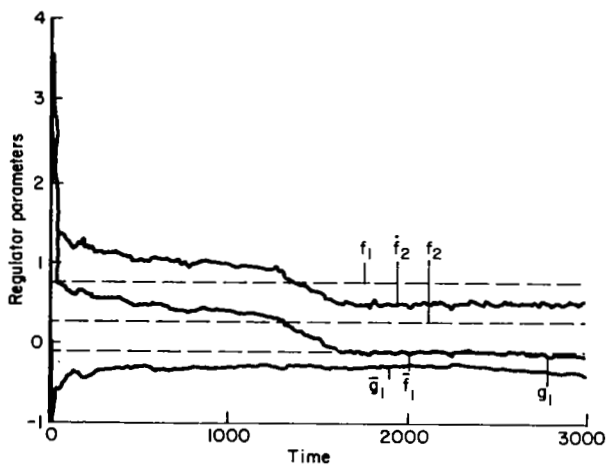


Figure 6.- Example regulator design using least squares and linear-quadratic estimation and control.

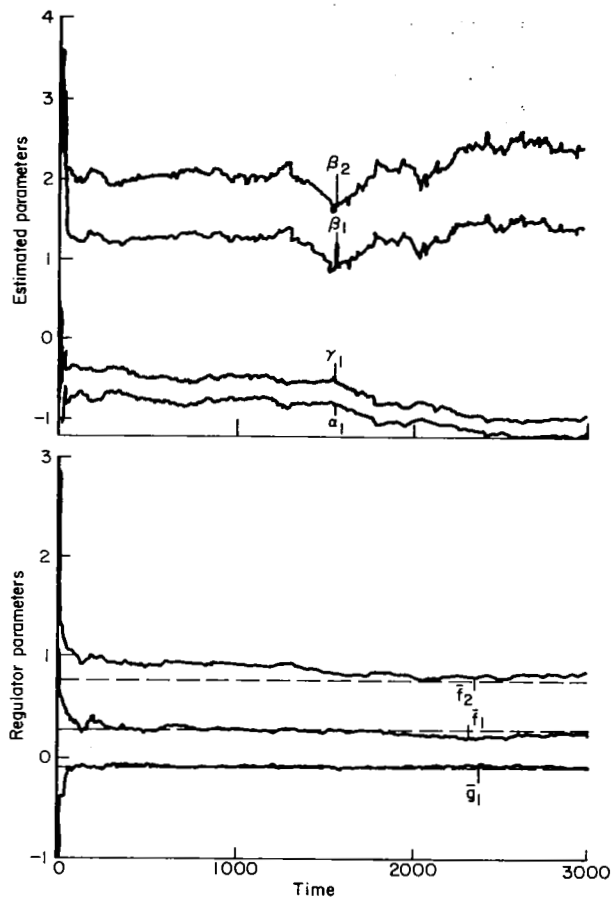


Figure 7.- Regulator design using extended least squares and linear-quadratic estimation and control.



# FAILURE DETECTION AND CONTROL-SYSTEM RECONFIGURATION:

## PAST, PRESENT, AND FUTURE

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### SUMMARY

The history of failure detection and redundancy management in aircraft applications is reviewed. To date, techniques related to that subject have been based mainly on hardware duplication of like components with failure monitoring and switchover or averaging for redundancy management. Specific examples of these techniques are discussed as they have been applied to the NASA F8 Digital Fly-by-Wire (DFBW) aircraft and are to be applied to the space shuttle vehicle in the near future.

Recently, interest has arisen in new mathematical developments that promise optimal failure detection and control law reconfiguration through the use of system dynamical equations and measurements from dissimilar components. Practical use of those developments depends on the availability of advanced logic processing such as is offered by modern high-speed computers. Recent results of simulations using those developments are presented. They indicate that, in the simulation environment, failure detection and control-system reconfiguration through the use of dissimilar components is possible and functions satisfactorily for sensors used for primary flight control. However, computational requirements for the system studied are large and exceed the capability of the F8-DFBW aircraft as presently configured. Due to the parallel nature of the algorithm needed for the system, the use of microprocessors as dedicated computational units should alleviate this problem. For the time being, however, to develop advanced redundancy management concepts to a technology application level, NASA is proceeding with the development of advanced suboptimal systems. Plans for a flight test of the advanced redundancy management system are presented and discussed.

### PAST

Since the advent of aviation there has been concern regarding safety of flight. That concern is most apparent in the aircraft flight-control system. The purpose of most flight-control systems is to translate motions from the pilot's controllers into appropriate motions of control surfaces located on the aircraft. In the early days of aviation, this was accomplished by an all-mechanical control system as indicated in figure 1(a). That type of system is in use today in most light aircraft. One feature of the mechanical

system is that loads required to move the control surfaces are transmitted through mechanical linkages to the pilot, thereby giving him a feel of the aircraft. As aircraft became larger, the forces transmitted also became larger. That led to the requirement for the fully powered hydraulic system shown in figure 1(b). The forces applied to the control surfaces are not felt by the pilot in the fully powered system so that artificial feel and trim had to be used. Also, because of dependence on the actuation system, increased reliability was needed. This led to tandem actuators, multiple hydraulic supply systems, hydraulic voting mechanisms (usually force sum devices) and many other *ad hoc* procedures to meet the vehicle design requirements from the reliability standpoint.

Desire to have aircraft fly in conditions where its flying qualities are undesirable led to the development of the stability augmentation system (SAS) shown in figure 1(c). That system uses sensors to detect aircraft motions and electrical signals determined from the sensors to activate the hydraulic system. The signal is applied in addition to the control commands from the pilot's controls. The SAS system introduced new requirements for reliability since failure of the sensors could adversely affect the vehicle's motions. Again, these were resolved largely through *ad hoc* procedures usually involving limiting the authority of the SAS inputs. This has been done by a variety of methods usually employing ingenious mechanical devices (or nightmares, depending on your point of view). One approach to eliminate the complex, weighty, and maintainably undesirable mechanized nightmare is being developed by government and industry — the fly-by-wire control system (shown in fig. 1(d)). That system will employ electrical wires to replace the complex mechanical linkages indicated above. It enables a flexible design with advantages of weight saving and simplicity. The main reasons that such systems are not used in operational aircraft today is that there is a general lack of confidence in the basic reliability of the electrical and electronic components used in a fly-by-wire design. An increase in the basic reliability of the fly-by-wire system will be necessary before it can be used operationally.

NASA is involved in two programs to develop fly-by-wire technology. At the Dryden Flight Center (DFRC), a digital fly-by-wire system is being installed onboard an F8 aircraft as part of the F8-DFBW project. Indeed they are involved in the second phase of their activity, having completed a phase where the APOLLO digital computer was used as the central control unit in a fly-by-wire system. The current phase of the project involves replacing the APOLLO computer with a set of IBM AP-101 computers. Concerning reliability requirements for the basic fly-by-wire mechanization, it was determined that fly by wire required quadruply redundant secondary actuators. The actuator concept used in the first phase of their work is illustrated in figure 2 (ref. 1). The actuator is constructed with three tandem pistons on a common shaft. It is powered by two independent hydraulic supplies. It has four independent servosystems to control the position of the actuator. Normal operation uses servosystem 1, which consists of an active and a monitor, two-stage, flapper nozzle servo valve. The active valve is monitored for failure by the monitor valve using a hydraulic comparator. If the difference in output of the monitor and active unit exceeds a threshold, the supply pressure engaging the active valve will be dumped to return (see slide valve at

lower part of servosystem 1 block). This will occur with motion of the comparator spool in either direction. Simultaneously, the hydraulic engage valve for servosystem 3 is hydraulically enabled. The primary channel fail switch causes all remaining (servosystems 2 and 3) engage solenoids to be energized. Thus the actuator is configured as a force summed triple-tandem actuator when all engage valves are engaged. Additional failures will result in some degradation of performance.

The real question is: what is the cost of providing redundancy in the form of hardware duplication? Some measure of that is obtained from the actuator example. This resulted from the existence of ground simulators required for F8 project support. Because of the lack of a requirement for quadruplex actuators in the simulator environment, Langley Research Center procured simplex actuators. The cost of the quadruplex actuators is approximately \$20,000 per actuator, whereas simplex actuators are available at one-quarter the cost. In addition to increased cost, there also remains a question of performance degradation in redundant systems. When Langley acquired their actuators, it was determined that their bandpass had to be reduced from 50 to 10 Hz to simulate the quadruplex actuators onboard the aircraft.

#### PRESENT

The other NASA activity to develop fly-by-wire technology is the space shuttle project. Early in the project, it was determined that mission requirements for the shuttle dictated fly by wire. In addition to flight control requirements, the space shuttle has navigational reliability requirements that dictate advanced failure detection and redundancy management techniques not previously used. To meet reliability requirements, the shuttle avionics system evolved to a hardware configuration that uses hardware duplication for the different systems. For example, in hardware, the rate gyro assembly is duplicated three times and the computers five times. Management of the hardware redundancy for the components, a major problem, will be done by the computers. The computers operational flight program is divided into modules with all hardware interface handled in a module referred to as subsystem operations. That module acts as a system executive and controls all input/output interfaces. It also passes data from the I/O interface to the applications programs (guidance, navigation, flight control, etc.). A redundancy management module determines which data are to be passed to the remaining applications programs and to hardware. The techniques for doing that depend on the particular type of data being processed. Techniques used for sensor data, controller data, switch data, and actuator data tracking tests, builtin stimulus tests (BIST), and comparison tests. One unique feature related to redundancy management, not used in past aircraft applications, is the handling of the navigational data (fig. 3). Three inertial measurement units are used in parallel to generate position and velocity estimates. The position and velocity estimates are then adjusted for new navigation data input by the navigational sensors using a Kalman-Bucy-type filter. A data selection algorithm is then used to determine data to be passed on to the remaining applications programs, namely, the guidance and control modules.

Methods used by the shuttle system to select data depend on the level of data redundancy available. Candidate three-level methods are being considered which include simple average, sample middle select, Kaufman's weighted average, and Landley's method. Two-level methods being considered include averaging and voting using an alternate data source (i.e., using, say, NAV module generated pitch rate to determine which of two pitch rate gyros is functioning properly). In the two-level case, there was a possibility of including an advanced redundancy management algorithm based on early elements of decision theory. For the two-level case, a filter using Wald's sequential probability ratio test is being considered. Although it offers the potential of improved redundancy management (especially for small bias errors on IMU data), the method was computationally burdensome. It also relied heavily on the prior statistics determined from sensor bench tests.

In addition to the navigation module, there is, of course, a requirement for management of redundancy in the more classical fashion for the shuttle for the controls and sensors. One proposed method being evaluated for handling the sensor redundancy is: under no failures, middle value select will be used with tracking test to detect failures. Failure thresholds will be taken about the selected signal. A failure to track within the threshold is considered to be a failure of the device if it fails to track for N consecutive samples nominally spaced at 40-msec intervals. After the first failure, the failure is displayed to the crew and tracking tests are mechanized the same as in the no failure case, except that thresholds are taken about the average of the two good sensors. Again, failure to track assumes that a failure has occurred if it occurs for N consecutive samples. The action to be taken following a second failure is to notify the crew and safety select a single unit. BIST would then be initiated on the remaining unit. If it is determined to be a good unit, an interchange would then be made of the selected unit.

#### FUTURE

As is being applied in the shuttle project, one important potential for digital control systems is their ability to reorganize themselves following failures in sensors and actuators. For sensors, this has been accomplished using duplication of components and a simple voting process to manage the redundancy. Redundancy provided in that way can be termed "hardware" redundancy since it owes its existence to hardware duplication. Redundancy also exists between dissimilar components. However, to use that type of redundancy, one must be aware of the dynamic behavior of the system and the interactions between system components. Such redundancy is more properly termed "analytical" redundancy since it owes its existence to analytical knowledge of the systems behavior. Langley Research Center is developing this type of redundancy management for use in aircraft flight control. It is anticipated that a system will be developed and flight tested on the F8-DFBW aircraft at FRC. Simulation tests of an "analytical" redundancy management system for that aircraft have been made at Langley. The theoretical approach used and

portions of the results of those tests are described here. The approach is reported in detail in reference 2.

To use analytical redundancy requires that the designer have an analytical model that expresses the response of the system to control command inputs. It also requires consideration of the level of confidence that the designer has in the modeling process. Figure 4 shows the form of the model used for the F8 work. Basically, the model is a complete nonlinear six-degree-of-freedom model of the aircraft's motion which has a good kinematic representation using quaternions but uses an aerodynamic force representation that employs linear aerodynamic coefficient modeling. That is, the force and moment coefficients are linear functions of angle of attack and control positions. Also note that the F8-DFBW is a digitally controlled aircraft and hence requires a discrete form representation of the aircraft dynamics. That discrete form was obtained using Euler's method using a step size of 1/32 sec to be consistent with Langley's real-time computer system. The selection of the statistics was made considering that kinetic relations are more likely to be accurate than aerodynamic force and moment relations.

Figure 5 outlines the approach taken for failure detection. That approach is derived from Bayesian decision theory and involves selecting a group of hypotheses that are checked for validity. The different hypotheses represent different assumptions regarding the status of different subsystems. For example, hypothesis  $H_0$  might represent the assumption that all systems are operating normally. Hypothesis  $H_1$  might represent the hypothesis that a specific sensor has failed. Hypothesis  $H_2$  might represent the hypothesis that a multiple failure has occurred where, say, both the roll and yaw rate gyros have failed. The rationale for making a decision is to construct a function that assigns a cost to making an incorrect decision and then to minimize that function. The weights  $C_{ij}$  shown on the slide are the weights that the designer assigns to deciding on hypothesis  $H_j$  as true. Of course, if  $i = j$ , the decision is correct so that normally the terms of the form  $C_{ij}$  are taken to be zero. Figures 6 and 7 show simulation results that indicate the importance of the selection of the weighting coefficients. The results are generated from a simulation of an entry vehicle. Figure 6 shows the response of the vehicle with no augmentation on the left; on the right is the input response with augmentation. It is that response that we will look for in figure 7, which shows one continuous run of the simulation where different hypotheses are assumed and where a controller designed for the assumed hypothesis is used. In that run,  $H_0$  is the unfailed hypothesis,  $H_1$  is failure of the roll rate gyro,  $H_2$  is failure of the yaw rate gyro, and  $H_3$  is failure of an angle-of-sideslip indicator. During the entire run, the yaw rate gyro was failed by increasing the noise superimposed on its output. Note the character of the response when that hypothesis (i.e.,  $H_2$ ) is used.

The resolution to the Bayesian hypothesis testing problem involves two items (fig. 8). First one must construct hypothesis-conditioned estimates of the aircraft's state for each hypothesis selected. This implies having a bank of conditional mean estimators parallel in operation. The second step involves examination of the performance of each hypothesis-conditioned estimator considering the performance realizable if that hypothesis is used.

This is mechanized as a test of the residuals of each estimator with weights that depend on the expected error covariance of the estimates.

A block diagram of a failure detection system that might be implemented on an aircraft is indicated in figure 9. To implement the procedure requires computational capability in excess of that available on flight computers of today. Figure 10 lists some practical considerations used to study the concept. Simulation results to be presented were generated on the F8-simulation facility at Langley which involves a complete piloted six-degree-of-freedom simulation. The aerodynamic representation used in the simulation accounts for effects of Mach number, altitude, and aeroelasticity. Concerning the failure detection system, it used a nonlinear prediction model obtained using linear aerodynamics (as mentioned earlier). Filter gains for the different hypotheses-conditioned estimates were obtained using steady-state Kalman filter gains obtained at level flight equilibrium at 20,000-ft altitude and a Mach number of 0.6. Figure 11 shows the performance of that system when the simulated F8 aircraft is subject to a rapid roll maneuver. In these runs, the hypothesis tested were  $H_0$  - no failures,  $H_1$  - roll rate gyro failure, and  $H_2$  - yaw rate gyro failure. Traces of the actual and estimated failure state are shown on the slide.

NASA currently has a contract with the Draper Laboratory to mechanize and study requirements for an analytical redundancy management system relative to an F8-DFBW mechanization. It is anticipated that the output of that contract will lead to a flight test. The concept to be mechanized will be constrained by computer requirements and will hence probably not be a mechanization of the bank of filters approach being studied at Langley.

Systems employing banks of filters are items for the future. They will probably most easily be mechanized using dedicated microprocessors. Langley is initializing work on optimal decentralization of flight-control processes that use microprocessors as dedicated control units and signal processors. One experiment being considered is the construction of a microprocessor control unit for a digital actuator test facility at Langley. That control unit would provide for actuator stabilization, failure detection, and redundancy management. It would, in conjunction with other processors, provide an advanced redundancy management system using ideas presented here.

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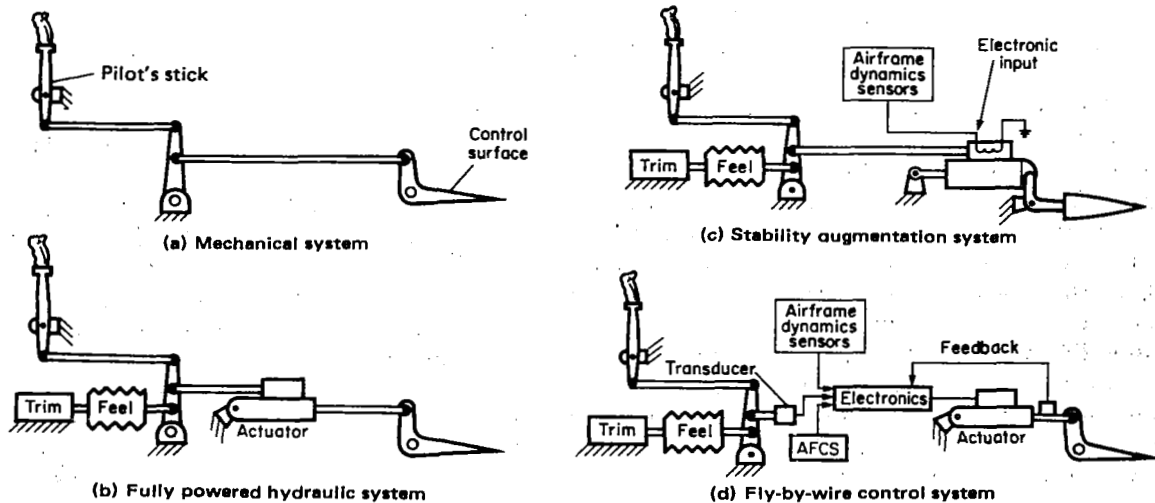


Figure 1.- Evolution of flight-control systems.

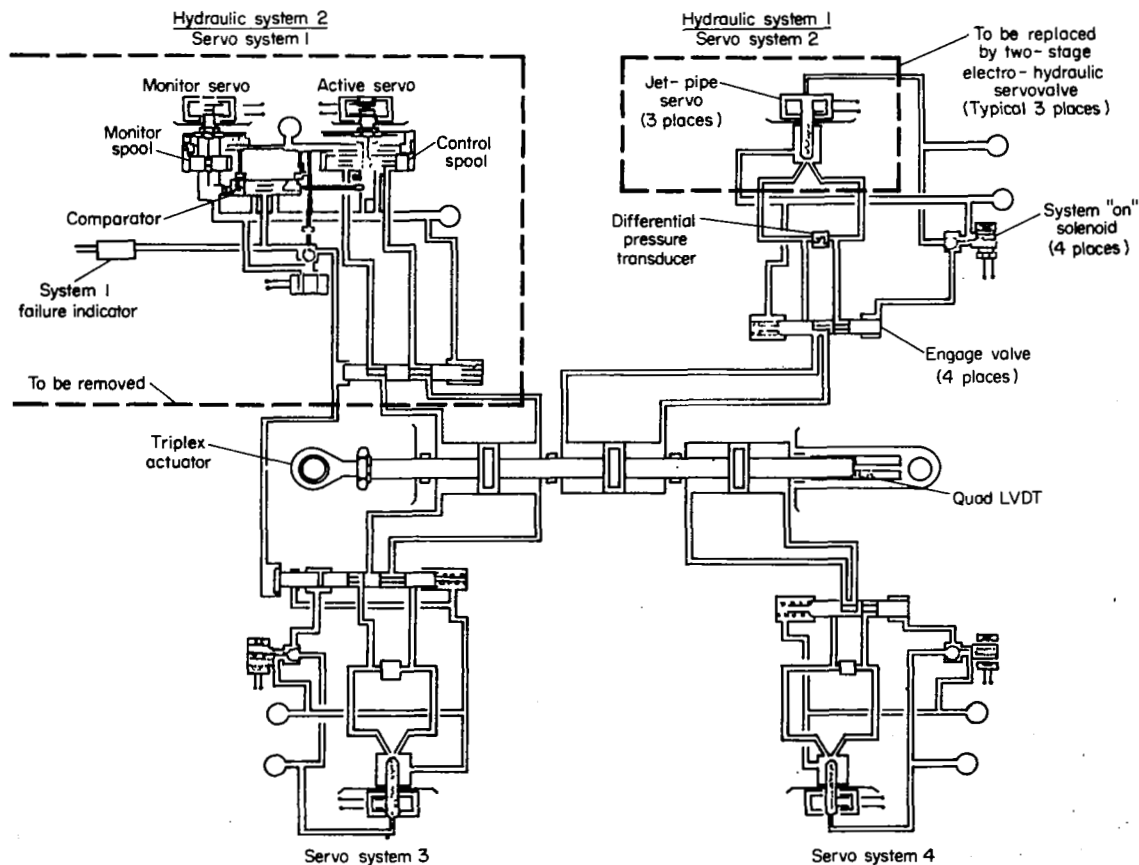


Figure 2.- Phase 1 quadruplex redundant secondary actuator for the F8 DFBW aircraft.



## HYPOTHESIS SELECTION

$$H_i: y(k) = h_i(x(k)) + v_i(k), \quad i = 0, 1, \dots, M$$

$$E(v_i) \text{ ASSUMED UNKNOWN, } \text{cov}[v_i(k)v_i(j)] = R_i \delta_{kj}$$

## BAYESIAN RISK FUNCTION

$$B = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} P_{H_j} C_{ij} \int_{Z_i} p_{y|H}(\alpha | H_j) d\alpha$$

$C_{ij}$  IS COST OF ACCEPTING  $H_i$  WHEN  $H_j$  IS TRUE

$Z_i$  IS THE DECISION REGION WHERE  $H_i$  IS ASSUMED TRUE

$P_{H_j}$  IS THE A PRIORI PROBABILITY OF  $H_j$

Figure 5.- Analytical redundancy management using multiple hypothesis testing.

1. REQUIRES A BANK OF  $M$  CONDITIONAL MEAN ESTIMATORS OPERATING IN PARALLEL
2. DECISION LOGIC IS BASED ON EXAMINATION OF RESIDUALS SELECTING THE HYPOTHESIS WHICH, ON-LINE, CORRESPONDS TO THE MINIMUM  $S_i$  WHERE

$$S_i = \frac{K}{2} \ln |Q_i| + \frac{1}{2} \sum_{j=1}^K r_i(j) Q_i^{-1} r_i(j)$$

$r_i$  - RESIDUAL OF THE ESTIMATOR FOR  $H_i$

$Q_i$  - MEASUREMENT ERROR COVARIANCE

$K$  - NUMBER OF SAMPLES IN DATA BASE

Figure 7.- Effects of accepting  $H_0$ ,  $H_1$ ,  $H_2$ , and  $H_3$  when  $H_2$  is true.

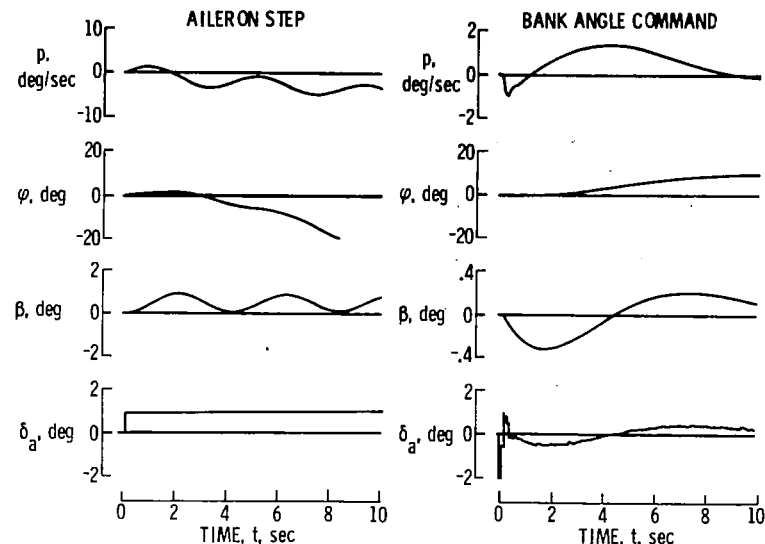


Figure 6.- Roll responses for simulated re-entry vehicle.

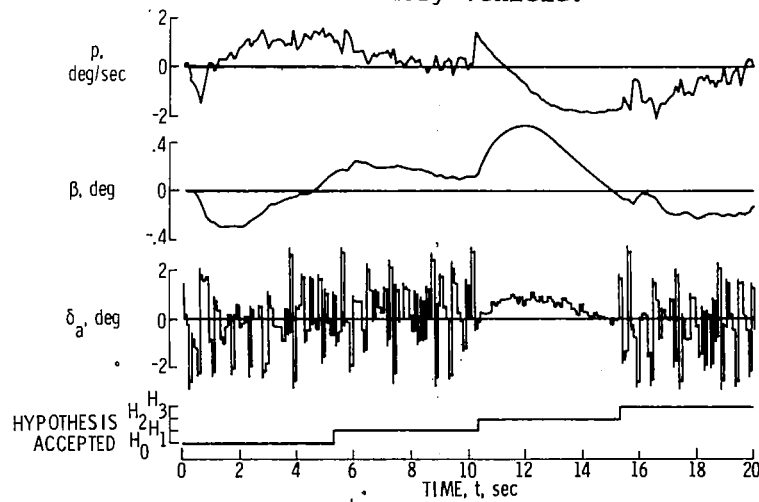
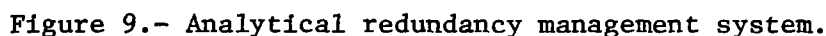


Figure 8.- Minimum Bayesian risk solution for analytical redundancy management.



CONSTANT FILTER GAINS AND ERROR COVARIANCES OBTAINED USING  
STEADY STATE KALMAN FILTER RESULTS.



# FACTS AND FICTION OF LEARNING SYSTEMS\*

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## 1. INTRODUCTION

The latest developments in computer technology and the ensuing successes of space exploration has generated a widespread belief that the aircraft and spacecraft of the future will be completely automated machines that may reliably accomplish preprogrammed treks and explorations in our galactic system.

It has been also widely assumed that modern optimal control theories provide the appropriate methodology to generate such automated controls. This may be so, but only in the far future when the computer technology would have progressed sufficiently more in sophistication and miniaturization. However, to meet the realistic demands of modern highly sophisticated high-speed aircraft and spacecraft requiring for their operation advanced system reliability, a man-machine interactive control system represents the optimal design. Such a system would require high-precision electromechanical controls as well as higher level decision-making and task coordination and planning, accepting only qualitative commands and task override by the human operator. Reliability of this nonconventional control system requires almost anthropomorphic functions for the controller such as training capabilities for the hardware for performance improvement in the presence of uncertainties, software optimal decision-making, and interface with the human operator.

The methodology that will provide the updated precision for the hardware control and the advanced decision-making and planning in the software control is called "learning systems and intelligent control." It has been developed theoretically as an alternative for the nonsystematic heuristic approaches of artificial intelligence experiments and the inflexible formulation of modern optimal control methods. Its basic concepts are discussed in the sequel and some feasibility studies of some practical applications are presented.

## 2. LEARNING SYSTEMS, SELF-ORGANIZING, AND INTELLIGENT CONTROL

The concept of "learning" has been used in system design and control whenever the *a priori* information of the underlying process is uncertain or unknown (ref. 1). This concept may be applied with equal effectiveness to identify uncertain parameters or improve directly the performance of the

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system by appropriate higher level decision-making which is sequentially updated by previous experience.

The following definitions regarding learning systems and learning control systems have been formulated by a subcommittee of Control Systems Society of IEEE and are presented here for completeness (ref. 2).

*Definition 1.* A system is called *learning* if the information pertaining to the unknown features of a process or its environment is acquired by the system, and the obtained experience is used for future estimation recognition, classification, decision or control such that the performance of the system will be improved.

*Definition 2.* A learning system is called a *learning control system* if the acquired information is used to control a process with unknown features.

Note that learning systems represent a wide class of processes, wider than control systems. As such they may serve as high-level decision-makers, problem solvers, and task planners and organizers in artificially intelligent systems like autonomous robots, interactive man-machine systems, bionic devices, and pattern recognizers. Such learning systems that may perform *on-line* as well as *off-line* are characterized as *intelligent systems or control* and are intended for highly sophisticated anthropomorphic functions.

When learning is applied to control hardware systems, one may devise a controller that would *on-line* improve its performance regardless of uncertainties governing its mathematical modeling. Such control systems have been characterized as *self-organizing control systems* and are defined in reference 2 as follows:

*Definition 3.* A control process is called *self-organizing* if reduction of the *a priori* uncertainties pertaining to the effective control of the process is accomplished through information accrued from subsequent observations of the accessible inputs and outputs as the control process evolves. A controller structured to perform the above task *on-line* is called a *self-organizing controller*.

*Definition 4.* A self-organizing control process is called *parameter-adaptive* if it is possible to reduce the *a priori* uncertainties of a parameter vector characterizing completely the process through subsequent observations of the accessible inputs and outputs of the system as the control process evolves. A controller structured to perform the above task *on-line* is called a *parameter-adaptive S.O. controller*.

*Definition 5.* A self-organizing control process is called *performance-adaptive* if it is possible to reduce directly the uncertainties pertaining to the improvement of the performance

of the process through subsequent observations of the accessible inputs and outputs of the system as the control process. A controller structured to perform the above task *on-line* is called a *performance-adaptive S.O. controller*.

It is obvious from the previous discussion that learning systems cover a wide area of decision-makers, ranging from highly sophisticated intelligent machines to advanced control systems. Generally, however, they deal with complex processes for which simple classical control techniques cannot be applied. Actually, this is one of the reasons why learning systems have not been popular with scientists and engineers until recently. The processes under their consideration have been too simple and too well defined to require any advanced decision-making.

In the last few years, the study of large-scale systems in "soft sciences," energy environment, space technologies, etc., have found the existing analytic tools inadequate. Therefore, scientists started looking for new methodologies, one of them being "learning systems," which provide a new dimension in the various control problems. In particular, in aircraft and space technology with the new high-speed sophisticated aircraft and space vehicles operating in unknown environment, two main reasons prompted the use of self-organizing control as a learning system for reliable operation: the uncertainties involved with the mathematical model and the need of lower-order models to represent high-dimensional systems.

Several feasibility studies have proven the advantages of such controllers and certain actual tests are already in progress by Honeywell and will be reported in these proceedings.

A review of the most successful self-organizing control methods is presented in the next section, while two feasibility studies performed by the author and his colleagues are discussed in the sequel. However, a word of caution is appropriate here; learning systems and self-organizing controls should not be used to drive systems for which a simple controller designed by classical methods is adequate. Sophisticated techniques are needed only when complex systems require them.

### 3. A REVIEW OF SOME SELF-ORGANIZING CONTROL METHODOLOGIES

Numerous methods that qualify under self-organizing control definitions have appeared in the literature since 1950. Most of these methods have appeared in expository publications (refs. 3-17) under various names such as adaptive, optimalizing, self-optimizing, etc., systems. They may be subdivided into two categories, the first one treating systems operating in a deterministic environment and the other for systems operating in a stochastic environment. The first category covers systems like Margolis and Leondes' parameter tracking servo, the polynomial expansion methods, the automatic spectrum analyzer approach, the functional analysis approach, the model reference adaptive control methods, error correcting methods, and many others (ref. 12).

The second category, which is the most realistic of the two since it incorporates in the formulation the irreducible uncertainties introduced by the environment, covers a collection of algorithms with various degrees of uncertainties about the plant dynamics. In the subcategory of parameter-adaptive self-organizing (SO) controls (ref. 12), one may have to select among the linearized stochastic optimal-control algorithms (ref. 18), the open-loop feedback optimal control algorithms (refs. 19 and 20), the Bayesian learning algorithms (refs. 21-23), the parallel identification and control (refs. 24 and 25), and the actively adaptive control algorithm (ref. 26). The characteristic of this subclass is that a parameter identification is necessary to provide the information about the uncertainties of the process. In the subclass of performance adaptive SO controls, the choice may be made among the crosscorrelations approach (refs. 27 and 28), stochastic automata methods (refs. 29 and 30), the stochastic approximation algorithm (refs. 31 and 32), the expanding subinterval algorithm (ref. 33), etc. The characteristic property of this subcategory is that the search for the reduction of uncertainties pertains to the evaluation of the performance criterion which indicates the improvement of the performance of the systems. Review monographs that give a detailed account of the formulation of these algorithms have recently appeared in the control literature (refs. 7 and 12), and several applications in man-machine and robotic systems as well as spacecraft control (refs. 19 and 34) have been discussed. Since the space is rather limited here, no detailed account is given and the interested reader is referred to the literature. Instead, two space vehicle studies, utilizing SO control techniques and being produced by the author and his colleagues as feasibility studies, are presented in the sequel in terms of simulation results.

#### 4. PARAMETER-ADAPTIVE SO CONTROL FOR BOOSTER STAGE OF A SPACE VEHICLE

Parallel identification estimation and control is used as a typical parameter-adaptive SO control scheme to demonstrate the feasibility and application of the method to a booster stage of a Saturn space vehicle (ref. 24). In this application, the goal is the simultaneous flight parameter identification and control of the vehicle during its mission, that is, from launch to stage separation while operating in a stochastic environment, generated by external wind gusts and control input disturbances.

Considering a pitch plane motion only, the booster is shown diagrammatically in figure 1, where  $\alpha$  is angle of attack (deg),  $\theta$  is attitude angle (deg), and  $\delta$  is gimbal angle of the rocket engine thrust chambers (deg). All of the above variables are perturbation angles from a reference trajectory. Thus the control problem will be one of maintaining small deviations about a nominal flight path. For small-angle variations and other simplifying assumptions, the rigid-body equation of motion and the dynamics of the engine actuator are given, respectively, by

$$\frac{d^2\theta}{dt^2} - \mu_\alpha \theta = \mu_\delta \delta \quad (1)$$



$$\frac{d\delta}{dt} + k_C \delta = \delta_C k_C \quad (2)$$

where  $\mu_\alpha$  and  $\mu_\delta$  are unknown time-varying flight parameters and  $\delta_C$  is the gimbal command (in deg) and  $1/k_C$  is the engine thrust chamber gimbaling time constant. As it stands, the rigid-body equation of motion in equation (1) represents an aerodynamically unstable vehicle. Therefore, a stabilization autopilot is initially synthesized using "worst-case" flight parameters  $\mu_\alpha$  and  $\mu_\delta$  to ensure stability of the uncontrolled system throughout its flight profile. Generally, the values for the worst-case flight parameters are known from prior launch data. Figure 2 is a block diagram of the engine actuator-vehicle dynamics and autopilot gains. The constants  $k_A$ ,  $k_R$ , and  $k_C$  are chosen to ensure stability of the overall system for worst-case flight parameters. The variable  $\theta_C$  represents the attitude command input and is used to control the state of the system.

Letting  $q = \dot{\theta}$ , the system in figure 2 can be written in state variable form as

$$\begin{bmatrix} \dot{\theta} \\ \dot{q} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \mu_\alpha & 0 & \mu_\delta \\ -k_C k_A & -k_A k_R k_C & -k_C \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ k_A k_C \end{bmatrix} (\theta_C + y + w) \quad (3)$$

where  $\theta_C$  represents the feedback control input. The newly introduced terms  $y$  and  $w$  correspond to the perturbation noise input and system disturbance input, respectively. Using attitude and rate gyros, the output measurement equation for the system in equation (3) is given by

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \delta \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (4)$$

The two output measurement signals are therefore contaminated by additive noise disturbances to account for external wind gusts.

Using the numerical values in reference 24 and a sampling time of 0.005 sec, the system is converted to the discrete-time form:

$$\begin{bmatrix} \theta(k+1) \\ q(k+1) \\ \delta(k+1) \end{bmatrix} = \begin{bmatrix} 1.0009 & 0.0049794 & 0.002213 \\ 0.29102 & 0.98778 & 0.8546 \\ -0.48792 & -0.028015 & 0.8052 \end{bmatrix} \begin{bmatrix} \theta(k) \\ q(k) \\ \delta(k) \end{bmatrix} + \begin{bmatrix} 0.0003998 \\ 0.23582 \\ 0.48036 \end{bmatrix} [\theta_C(k) + y(k) + w(k)] \quad (5)$$

$$z(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta(k) \\ q(k) \\ \delta(k) \end{bmatrix} + v(k) \quad (6)$$

where

$$z(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} \quad \text{and} \quad v(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix}$$

The random variables  $y(k)$ ,  $w(k)$ , and  $v(k)$  are assumed to be jointly independent and have Gaussian distributions with the statistical properties:

$$\left. \begin{aligned} E[y(k)] &= E[w(k)] = E[v(k)] = 0 \\ E[y(k)y(j)] &= 9\delta_{kj} \\ E[w(k)w(j)] &= 0.01\delta_{kj}, \quad E[v(k)v^T(j)] = \begin{bmatrix} 0.01\delta_{kj} & 0 \\ 0 & 0.01\delta_{kj} \end{bmatrix} \end{aligned} \right\} \quad (7)$$

$k, j = 0, 1, 2, \dots$

For this problem, the performance criterion to be minimized is given by

$$J(u) = E \frac{1}{3000} \sum_{i=0}^{3000} \|x(i+1)\|_Q^2 + r\theta_C^2(i) \quad (8)$$

where

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}; \quad r = 10$$

Saturation bounds on  $\theta_C$  were set at  $\pm 5^\circ$  to ensure that the system variables would not become unrealistically large during the period of identification.

To set the matrices in a form suitable for identification, the following transformation is necessary for which the controllability and observability subindices are

$$p_1 = 1, \quad p_2 = 2, \quad q_1 = 3 \quad (9)$$

Using the selection procedure in equation (6), define a matrix

$$P = \begin{bmatrix} h_1^T \\ h_2^T \\ h_2^T A \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.291 & 0.9877 & 0.8546 \end{bmatrix} \quad (10)$$

Through the following change of coordinates

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = P \begin{bmatrix} \theta(k) \\ q(k) \\ \delta(k) \end{bmatrix} \quad (11)$$

the system (eqs. (3) and (4)) is transformed into the canonical form:

$$x(k+1) = A_0 x(k) + B_0 [\theta_C(k) + y(k) + w(k)] = \left[ \begin{array}{c|cc} \overbrace{1}^{P_1} & \overbrace{0.002419 \quad 0.0026}^{P_2} \\ \hline 0 & 0 & 1 \\ -0.3591 & -0.8183 & 1.7946 \end{array} \right] x(k) + \begin{bmatrix} 0.00039986 \\ 0.23582 \\ 0.643 \end{bmatrix} [\theta_C(k) + y(k) + w(k)] ; \quad m = 2$$

$$z(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} = H_0 x(k) + v(k) \quad (12)$$

The parallel identification, estimation, and control scheme is presented in figure 3. Observing that system (10) is completely controllable and completely observable and that if the random variables  $y(k)$ ,  $w(k)$ , and  $v(k)$  have finite moments up to the fourth order, the identifier will produce estimates on the matrices  $A_0$  and  $B_0$  of system (1) by use of a stochastic approximation algorithm such as

$$\left. \begin{aligned} \hat{\theta}_i(k+g) &= \hat{\theta}_i(k-1) + \gamma \left( \frac{k-1}{g+1} \right) \frac{Y^{(k+g-1)}(k)}{Y^{(k)}(k)} [z_i(k+g) - Y^{T(k+g-1)}(k) \hat{\theta}_i(k-1) \\ &\quad - U^{T(k+g-1)} \hat{\theta}_i(k-1)] , \\ k &= 1, \quad g+2, \quad 2g+3, \dots; \quad i = 1, 2, \dots, m \end{aligned} \right\} \quad (13)$$

where

$$\theta_i^T = \left[ h_i^O{}^T B_O, h_i^O{}^T A_O B_O, \dots, h_i^O{}^T A_O^{g-1} B_O \right]; \quad i = 1, 2, \dots, m$$

$$\beta_i^T = \left[ h_i^O{}^T D_O, h_i^O{}^T A_O D_O, \dots, h_i^O{}^T A_O^{g-1} D_O \right]; \quad D_O \equiv B_O$$

$$Y_{(k)}^T(k+g-1) = [y^T(k+g-1), y^T(k+g-2), \dots, y^T(k)]$$

$$U_{(k)}^T(k+g-1) = [u^T(k+g-1), u^T(k+g-2), \dots, u^T(k)]$$

$$W_{(k)}^T(k+g-1) = [w^T(k+g-1), w^T(k+g-2), \dots, w^T(k)]$$

The term  $H_O$  in equation (10) has been partitioned as

$$H_O^T = [h_1^O, h_2^O, \dots, h_m^O]$$

and  $\hat{\theta}_i(0)$  is chosen arbitrarily. For convergence of equation (11),

$$\left. \begin{aligned} \lim_{i \rightarrow \infty} \gamma(i) = \infty, \quad \sum_{i=1}^{\infty} \gamma(i) = \infty, \quad \sum_{i=1}^{\infty} \gamma^2(i) < \infty \\ E[\|\hat{\theta}_i(0)\|^2] < \infty, \quad i = 1, \dots, m \end{aligned} \right\} \quad (14)$$

The original parameters are recovered by the following general transformations used here for  $m = 2$ :

$$B_O = \left[ \begin{array}{ccc} \theta_1^1 & \theta_1^2 & \dots & \theta_1^r \\ \vdots & & & \vdots \\ (p_1-1)r+1 & & & p_1 r \\ \theta_1^1 & \dots & & \theta_1^r \\ \hline \theta_2^1 & & \dots & \theta_2^r \\ \vdots & & & \vdots \\ (p_2-1)r+1 & & & p_2 r \\ \theta_2^1 & \dots & & \theta_2^r \\ \hline \vdots & & & \vdots \\ \hline \theta_m^1 & \theta_m^2 & \dots & \theta_m^r \\ \vdots & & & \vdots \\ (p_m-1)r+1 & & & p_m r \\ \theta_m^1 & \dots & & \theta_m^r \end{array} \right] \left. \begin{array}{l} \left. \begin{array}{c} \left. \begin{array}{c} \theta_1^1 \\ \vdots \\ (p_1-1)r+1 \\ \theta_1^1 \end{array} \right\} \right. \\ \left. \begin{array}{c} \theta_2^1 \\ \vdots \\ (p_2-1)r+1 \\ \theta_2^1 \end{array} \right\} \\ \vdots \\ \left. \begin{array}{c} \theta_m^1 \\ \vdots \\ (p_m-1)r+1 \\ \theta_m^1 \end{array} \right\} \end{array} \right\} \begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_m \end{array} \end{array} \right\} \quad (15)$$

$$\begin{aligned}
A_0 = & \begin{bmatrix} \theta_1^{r+1} & \dots & \theta_1^{q_1 r+1} & \theta_1^{r+2} & \dots & \theta_1^{q_r r+r} \\ \theta_1^{2r+1} & \dots & \theta_1^{(q_1+1)r+1} & \theta_1^{2r+2} & \dots & \theta_1^{q_r r+2r} \\ \vdots & & \vdots & \vdots & & \vdots \\ \theta_1^{p_1 r+1} & \dots & \theta_1^{(q_1+p_1-1)r+1} & \theta_1^{p_1 r+2} & \dots & \theta_1^{(q_r+p_1-1)r+r} \\ \theta_2^{r+1} & \dots & \theta_2^{q_1 r+1} & \theta_2^{r+2} & \dots & \theta_2^{q_r r+r} \\ \vdots & & \vdots & \vdots & & \vdots \\ \theta_m^{p_m r+1} & \dots & \theta_m^{(q_1+p_m-1)r+r} & \theta_m^{p_m r+2} & \dots & \theta_m^{(q_r+p_m-1)r+r} \end{bmatrix} \times \\
& \begin{bmatrix} \theta_1^1 & \dots & \theta_1^{(q_1-1)r+1} & \theta_1^2 & \dots & \theta_1^{q_r r} \\ \theta_1^{r+1} & \dots & \theta_1^{q_1 r+1} & \theta_1^{r+2} & \dots & \theta_1^{q_r r+r} \\ \vdots & & \vdots & \vdots & & \vdots \\ \theta_1^{(p_1-1)r+1} & \dots & \theta_1^{(q_1+p_1-2)r+1} & \theta_1^{(p_1-1)r+2} & \dots & \theta_1^{(q_r+p_1-1)r} \\ \theta_2^1 & \dots & \theta_2^{(q_1-1)r+1} & \theta_2^2 & \dots & \theta_2^{q_r r} \\ \vdots & & \vdots & \vdots & & \vdots \\ \theta_m^{(p_m-1)r+1} & \dots & \theta_m^{(q_1+p_m-2)r+1} & \theta_m^{(p_m-1)r+2} & \dots & \theta_m^{(q_r+p_m-1)r} \end{bmatrix}^{-1}
\end{aligned} \quad (16)$$

A quick glance shows that the above algorithms are dependent on knowledge of the statistics of the noises  $w(k)$ ,  $v(k)$ ,  $y(k)$ , or of the matrix  $D_0$ . The structure of the feedback controller can also attain an arbitrary form; in fact, it could be made dependent on the past of the identification estimates in equation (14).

The asymptotically optimal controller designed in parallel of the identifier is given by the following. The resulting form of the controller consists of a cascaded discrete-time Kalman filter and deterministic control gain, both of these elements using estimates of the unknown system matrices from an identifier. Thus the controller structure attains the following simple form:

$$u(k) = K^\infty(k) \hat{x}(k) \quad (17)$$

where  $K^\infty(k)$  is the steady-state deterministic optimal gain at time  $k$  and is obtained by a backward iterative procedure on the discrete-time version of the matrix Riccati equation

$$\left. \begin{aligned} K_i(k) &= -[\hat{B}_o^T(k) W_{i+1} \hat{B}_o(k) + R]^{-1} \hat{B}_o^T(k) W_{i+1} \hat{A}_o(k) \\ W_i &= \hat{A}_o^T(k) W_{i+1} \hat{A}_o(k) + \hat{A}_o^T(k) + W_{i+1} \hat{B}_o(k) K_i(k) + Q \\ i &= N-1, N-2, \dots; W_N = Q \end{aligned} \right\} \quad (18)$$

Estimates of the unknown system matrices from the identifier  $\hat{A}_o(k)$ ,  $\hat{B}_o(k)$  are used in the above equations in place of  $A_o$ ,  $B_o$ . At any time  $k$ , the stationary value of  $K_i(k)$  that results by iterating backward with respect to  $i$  is designated  $K^\infty(k)$ .

The state estimate  $\hat{x}(k)$  in equation (15) is obtained using a discrete-time Kalman filter and again uses estimates of the unknown system matrices in place of  $A_o$ ,  $B_o$ . The equations for the filter are then given by

$$\begin{aligned} \hat{x}(k+1) &= \hat{A}_o(k) \hat{x}(k) + \hat{B}_o(k) [u(k) + y(k)] + K_F(k+1) \left( z(k+1) \right. \\ &\quad \left. - H_o \{ \hat{A}_o(k) \hat{x}(k) + \hat{B}_o(k) [u(k) + y(k)] \} \right); \hat{x}(0) = 0 \end{aligned} \quad (19)$$

and

$$K_F(k+1) = P(k+1|k) H_o^T [H_o P(k+1|k) H_o + \sigma^2 I]^{-1}$$

$$P(k+1|k) = A_o(k) P(k|k) A_o^T(k) + q^2 D_o D_o^T$$

$$P(k+1|k+1) = [1 - K_F(k+1) H_o] P(k+1|k), P(o/o) \text{ specified}$$

where

$$E[w^T(k)w(j)] = q^2 I \delta_{kj}$$

$$E[v^T(k)v(j)] = \sigma^2 I \delta_{kj}$$

A saturation-type nonlinearity is inserted in the feedback loop to induce a stabilizing effect on the overall system and to more naturally depict the characteristics of all physical feedback controllers (i.e., they inherently saturate at a certain level).

Simulation of the system in figure 3 using the above controller was performed on a CDC-6500 digital computer. The results of the identification are presented in figure 4 and the performance cost results in figure 5. In these figures, the average normalized error represents a 15-run ensemble average and hence is an approximation to the mean-square normalized error. It is apparent after 10,000 iterations that the identification is consistent. For a sampling interval of 0.005 sec, this corresponds to a real time of 50 sec which represents about 1/3 of the total flight time. To shorten the identification

interval, 0.001 sec/iteration is achievable with present-day general purpose, onboard digital computers (for more details, see ref. 24).

## 5. PERFORMANCE ADAPTIVE SO CONTROL FOR A SPACE ATTITUDE CONTROL SYSTEM

The feasibility of application of the expanding subinterval performance adaptive SO control method has been investigated in conjunction with a single-axis attitude control problem of an orbiting satellite with random switching delay (ref. 35). The method was selected because it yields a global minimum of accrued performance, it is easy to implement, and does not depend on the knowledge of the system or the noise dynamics, thus generating a reliable algorithm for control of processes with extremely uncertain dynamics. In the particular case of attitude control, the problem may be interpreted as a double-integral plant with equations of motion:

$$\left. \begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 2.5 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi(t) \\ \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \eta_1(t) \\ \eta_2(t) \end{bmatrix} \\ \Omega_u &= \{|u| \leq 1\} \\ J(u) &= \lim_{t_f \rightarrow \infty} E \frac{1}{t_f - t_0} \left\{ \int_{t_0}^{t_f} [||x(t)||^2 + 0.1|u(t)|] dt \right\} \end{aligned} \right\} \quad (20)$$

The initial condition  $x(t_0)$  was assumed to be zero, and the random variables  $\xi(t)$  and  $\eta(t)$  were assumed to be Gaussian with statistics:

$$E[\xi(t)] = 0; \quad \text{cov} [\xi(t); \xi(\tau)] = \delta(t - \tau)$$

$$E[\eta(t)] = 0; \quad \text{cov} [\eta(t); \eta(\tau)] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \delta(t - \tau)$$

The form of the SO controller is a simplified version of the dead-zone controller obtained in reference 35 for a general stochastic fuel regulator problem of which this is a special case (see fig. 6):

$$u^*(\hat{x}, t) = -\text{DEZ}\{\hat{\omega}^*[\hat{x}(t), c, t]\} \quad (21)$$

where  $\hat{\omega}^*[\hat{x}(t), c, t]$  is the predicted value of the approximate switching function  $\omega^*[\hat{x}(t), \alpha, t]$ , included to minimize the influence of the random switching

delay with unknown statistics caused by component imperfections. The two functions are defined as

$$\omega^*[\hat{x}(t), \alpha, t] = \begin{bmatrix} \alpha_{11} & \dots & \alpha_{1s} \\ \vdots & & \vdots \\ \alpha_{m1} & \dots & \alpha_{ms} \end{bmatrix} \begin{bmatrix} \phi_1[\hat{x}(t), t] \\ \vdots \\ \phi_s[\hat{x}(t), t] \end{bmatrix} \quad (22)$$

$$\hat{\omega}^*[\hat{x}(t), c, t] = \begin{bmatrix} \beta_{11} & \dots & \beta_{1v} \\ \vdots & & \vdots \\ \beta_{m1} & \dots & \beta_{mv} \end{bmatrix} \begin{bmatrix} \psi_1(\omega^*, t) \\ \vdots \\ \psi_v(\omega^*, t) \end{bmatrix} \quad (23)$$

The vector  $c$  of the unknown parameters  $\alpha, \beta$  of dimension  $m(s+v)$  is defined as

$$c^T = [\alpha_{11}, \dots, \alpha_{ms}, \beta_{11}, \dots, \beta_{mv}] \in \Omega_c \quad (24)$$

The functions  $\phi_i[\hat{x}(t), t]$ ,  $i=1, \dots, s$ ,  $\psi_j(\omega^*, t)$ ,  $j=1, \dots, v$  are properly chosen to satisfy the conditions for realizability of the expanding subinterval algorithm

$$\left. \begin{aligned} \phi(\hat{x}) &\triangleq [1, x_1(t), 2x_2(t)]^T \\ \psi(\hat{\omega}) &\triangleq [\hat{\omega}(\hat{x}), \dot{\hat{\omega}}(\hat{x})]^T \end{aligned} \right\} \quad (25)$$

Consequently,

$$c = [\alpha_{11}, \alpha_{12}, \alpha_{13}, \beta_{11}, \beta_{12}]^T \quad (26)$$

for which the parameter space  $\Omega_c$  was taken from real data for orbiting satellites to be

$$\Omega_c = \left\{ \begin{aligned} &\{c: 0.05 \leq \alpha_{11} \leq 0.5, |\alpha_{11}/\alpha_{12}| \leq 2, \\ &\alpha_{13} = \alpha_{12}, \beta_{11} = 1, \beta_{12} = 0\} \quad \text{(cases 1 and 2)} \\ &\{c: 0.05 \leq \alpha_{11} \leq 0.5, |\alpha_{11}/\alpha_{12}| \leq 2 \\ &\alpha_{13} = \alpha_{12}, \beta_{11} = 1, 0 \leq \beta_{12} \leq 0.5\} \quad \text{(case 3)} \end{aligned} \right\} \quad (27)$$

where cases 1 and 2 treat the problem without compensating the random switching delay (e.g.,  $\omega^* = \hat{\omega}$  in eqs. (20) and (21)), while case 3 does.



The random search was implemented by

$$c_{i+1} = \begin{cases} \rho_i, & J(c_i) - J(\rho_i) > 2\mu \\ c_i, & J(c_i) - J(\rho_i) \leq 2\mu \end{cases} \text{ at } T_i \quad (28)$$

where  $T_i$  is the expanding subinterval

$$T_i = t_i - t_{i-1}, \quad i=1,2,\dots,\infty \quad T_{i-1} \leq T_i \quad \forall i \text{ and } \lim_{i \rightarrow \infty} T_i = \infty \quad (29)$$

over which the following "subgoal" is defined:

$$J_i(c_i) = (t_i - t_{i-1})^{-1} \int_{t_{i-1}}^{t_i} \|\hat{x}(t)\|^2 + 0.1 |u_j(t)| dt \quad (30)$$

In equation (28),  $\rho_i$  is the  $i$ th sample value of a vector random variable defined at every subinterval  $T_i$  on  $\Omega_c = \{c/u_c(t) \in \Omega_u\}$ , with probability density function  $p(\rho) \neq 0$  for all  $\rho \in \Omega_c$ ;  $\mu$  is an arbitrary positive number. It has been shown in reference 33 that, if the performance cost accrued during the search is defined by

$$V(k) = \frac{\sum_{i=1}^k T_i E_{\xi, \eta} \{J(c_i)\}}{\sum_{i=1}^k T_i}, \quad k = 1, 2, \dots \quad (31)$$

then in the limit and under certain conditions of convergence (ref. 33),

$$\left. \begin{aligned} \lim_{k \rightarrow \infty} \text{Prob} [V(k) - I_{\min} < 3\delta] &= 1; \\ I(u_c^*) &= \min_c I(u_c) = I_{\min} < \infty \end{aligned} \right\} \quad (32)$$

An EAI-680 analog computer was used in the simulation. As expected, the system is very sensitive to changes in the switching delay. In fact, for time scaling beyond 1000:1, the computer switching tolerance and dynamic response are such that analog simulation is not feasible. On the other hand, the system is quite insensitive to the exact filter gain. To analyze the "transient" aspects, three cases of the proposed control were considered.

For cases 1 and 2, no attempt was made to compensate for the random switching delay. The difference in these two cases is in the form of the probability density  $p(\rho)$ . In case 1, the expected value of the "test-point"  $\rho(k)$  is the present value of the parameter vector  $\alpha(k)$ , while in case 2 a bias is included to reinforce the motion in a successful direction. Case 3

is similar to case 2, but it adds the "predictive" aspect to reduce the effect of the random switching delay. The experimental results are given in terms of an ensemble average in a normalized per iteration cost instead of  $V(k)$ , as defined in equation (29), because it is more meaningful during the transient phase.

The average per iteration cost and its standard deviations are plotted in figures 7 and 8. The statistics depicted in figures 1 and 2 are based on sample sets of 100 independent simulations. Each run was sufficiently long to establish the steady-state per iteration cost. In all runs the limiting accuracy was governed by the accuracy of reading the analog records. It is observed that both the average and standard deviations of the per iteration cost have converged, in a practical sense, within the first 100 iterations. Acceleration schemes, based on heuristic selection of the search variable  $\rho_1$  in equation (28) have been used in the simulation (ref. 35). Figure 9 demonstrates the steady-state responses of the system before and after the transient period.

## 6. RELIABILITY CONSIDERATIONS

The reliability problem in the design of feedback controllers for aircraft and spacecraft systems may be considered from a different aspect, in view of the previous discussion. Modern high-speed vehicles with uncertainties in their dynamic models operate in a random environment. Stochastic solutions (refs. 36 and 37) are based only mostly on average *a priori* information about the uncertainties and are usually poor approximations of the desired nonlinear and optimal solution (ref. 38). Such solutions are highly unreliable for practical design purposes. Considerable improvement in reliability may be obtained if the process uncertainties may be reduced through a learning scheme in the controller, designed to acquire asymptotically its optimal value. SO controls provide such controllers that are also easy to implement and require considerably smaller computers than the general stochastic solutions. The previously presented feasibility studies should provide sufficient arguments to support this statement. From other comparative studies (ref. 12), it was found that the parallel identification estimation and control algorithm is 6 times faster, requires 18 times less core memory, has a 30-percent lower value of the performance cost over a linearized approximation of the stochastic optimal with unknown parameters, is 3 times slower, has the same core memory requirements, and gives 4 times higher performance cost than the stochastic optimal with known parameters. The expanding subinterval algorithm designed for systems with almost unknown dynamics is rather slower than the parameter adaptive algorithm but compares favorably otherwise (ref. 12). Additional acceleration may further speed up the random search technique. The above results were obtained from simulations on a CDC-6500 digital computer.

## 7. DISCUSSION

In conclusion, improvement in the reliability of the control algorithms may be obtained by using learning techniques to reduce the uncertainties of the systems and obtain an asymptotically optimal solution in a stochastic environment. These ideas have already been explored with various degrees of success by various areas of modern large-scale systems, as bioengineering, socio-economics, environmental, man-machine systems, etc. It is appropriate to mention that learning system approaches are superior to classical and other modern control methods only when the system involved is complex, when there are uncertainties in the system that can be reduced by learning instead of using their average values, and if an optimal solution could be obtained if the model of the system was completely known. Then, by use of learning techniques, one obtains solutions that are more reliable asymptotically than otherwise. However, if the system is simple enough that a stochastic or dual optimal solution is available, such a method will produce an overdesigned controller with very poor engineering characteristics. For this reason, certain over-designed learning controllers have created some skepticism already about learning and SO control to engineering circles. This skepticism, although justifiable, should not be extended to the problems considered here.

The problem of designing a SO control is by no means completed in the two feasibility studies presented. Stability analysis safety specification, etc., are required for a reliable engineering design. Some of these problems are discussed in reference 12. Most of the problems encountered in the design of SO control systems are still under investigation. However, the purposes of this paper are to motivate interest and to inspire enough confidence in the researchers and development engineers in the area to use these ideas in modern aircraft and spacecraft technology.

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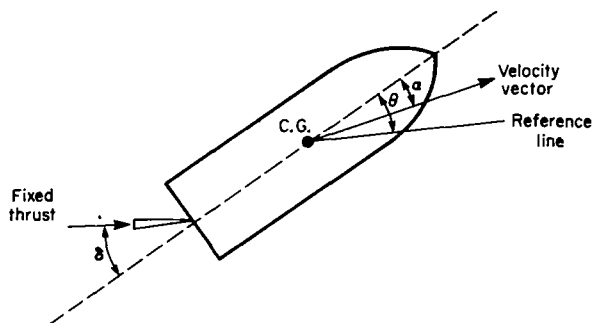


Figure 1.- Booster vehicle.

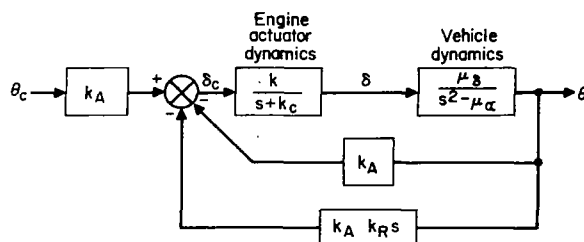


Figure 2.- Block diagram of uncontrolled system.

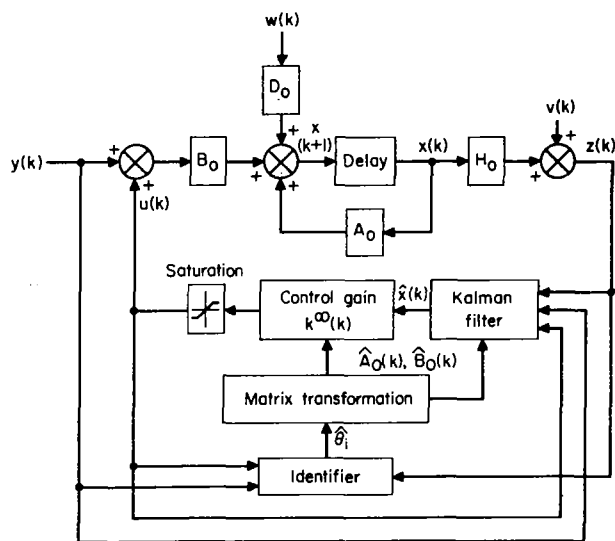


Figure 3.- Schematic diagram of the parallel identifications, estimation, and control.

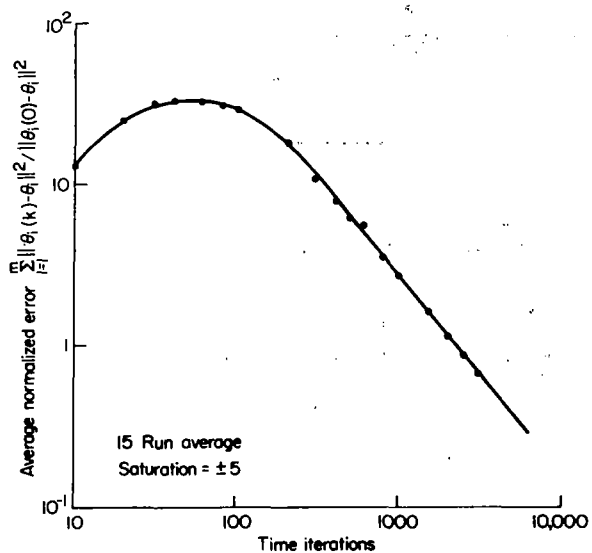


Figure 4.- Identification of the parameters of a third-order booster stage problem.

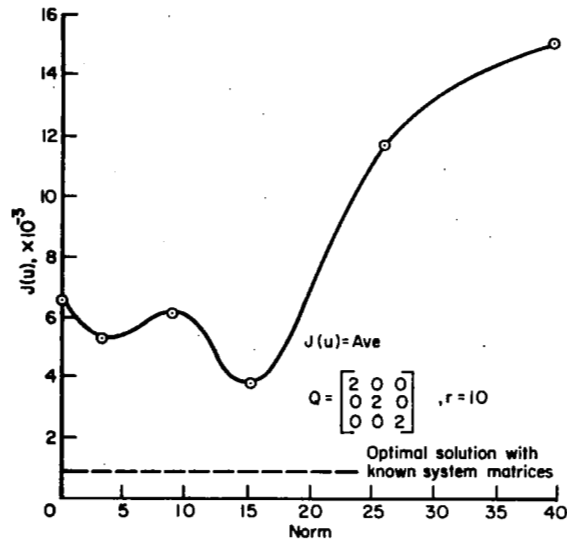


Figure 5.- Performance results — third-order booster stage problem.

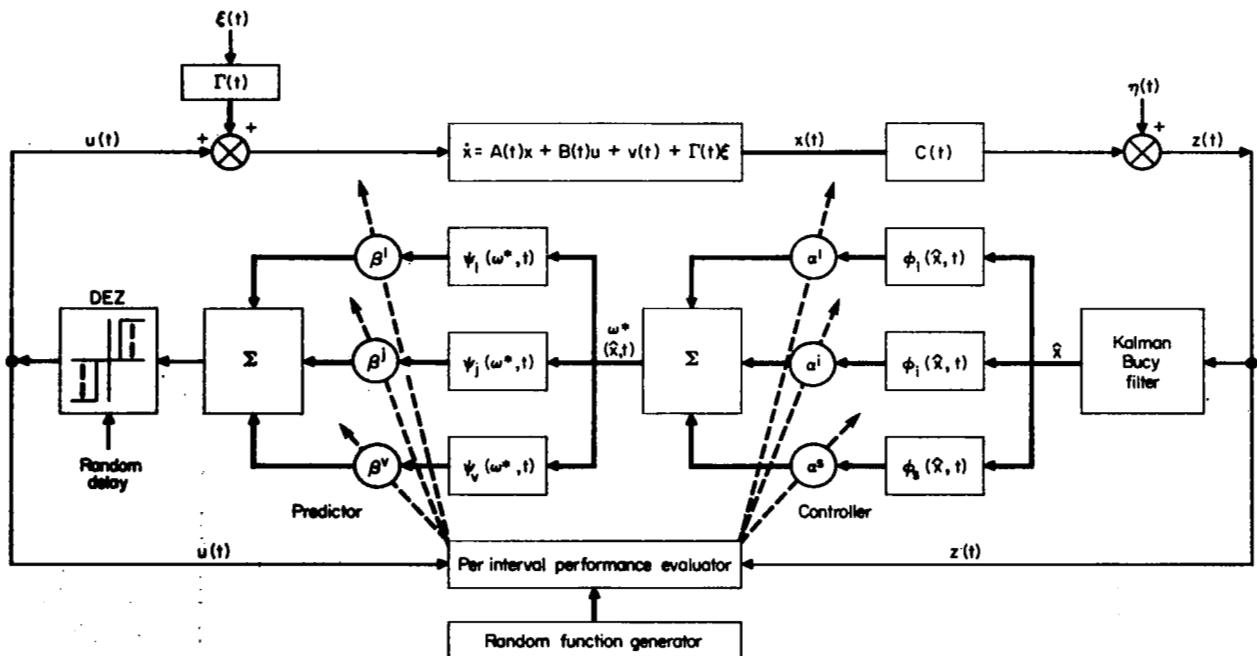


Figure 6.- Performance adaptive self-organizing control for the stochastic fuel regulator problem.

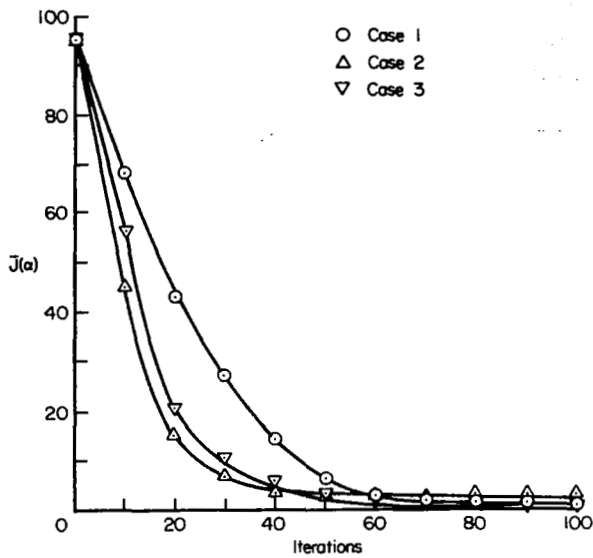


Figure 7.- Experimental results (average per iteration cost).

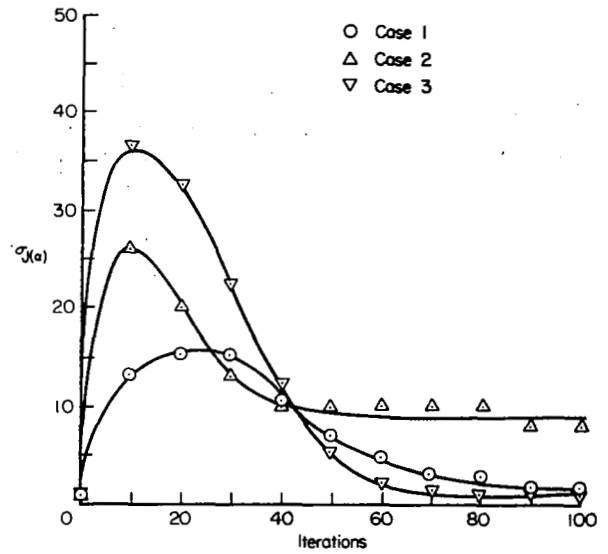
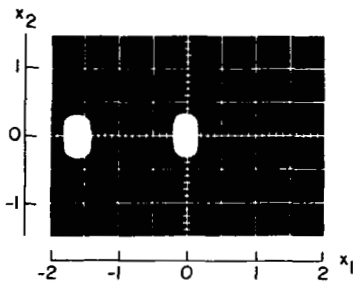
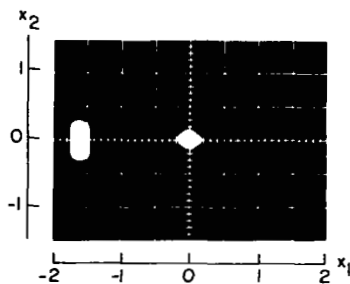


Figure 8.- Experimental results (standard deviation of per iteration cost).



(a). Case 1 and 2



(b). Case 3

Figure 9.- Steady-state responses before and after self-organization.

# STATUS OF DUAL CONTROL THEORY\*

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## I. INTRODUCTION

In many processes arising in engineering, economic, and biological systems, the problem of decision-making (or control) under various sources of uncertainties is inherent. These uncertainties prevent exact determination of the effect of all present and future actions, and therefore deterministic control theory is not applicable. If the effect of these uncertainties is small, one can still use optimal control theory to obtain a feedback control law based on deterministic considerations. The feedback nature of the control would tend to reduce the sensitivity to uncertainties but would require the state of the system to be measured exactly. Again, this assumption is good only when the measurement error is small in comparison with the signal being measured.

In many cases, the phenomena of uncertainty (including measurement error) can be appropriately modelled as stochastic processes, allowing them to be considered via stochastic optimal control theory. A very important concept in stochastic control is the *information pattern* available to a controller at a specific time, for the purpose of decision-making. As the process unfolds, additional information becomes available to the controller. This information may come about accidentally through past control actions or as a result of active probing which itself is a possible control policy. Thus "learning" is present, whether it is "accidental" or "deliberate." The information pattern available to the controller indicates not only what type of learning is possible at each instant of time, but, more importantly, whether future learning can be anticipated and how it could be influenced by present action (i.e., whether probing would be helpful in future learning). Since more learning may improve overall control performance, the probing signal may indirectly help in controlling the stochastic system. On the other hand, excessive probing should not be allowed even though it may promote learning because it is "expensive" in the sense that it will generally increase the expected cost performance of the system. This interplay between learning and control is the key issue of stochastic control theory.

This "dual" purpose of the control was pointed out by Fel'dbaum (ref. 1) using the stochastic dynamic programming approach (ref. 2). Because of this dual effect, Fel'dbaum called it "dual control theory." The subject was then studied by Aoki (refs. 3 and 4), Åström (ref. 5), Florentin (ref. 6), and

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Meier (ref. 7), via some simple two-stage examples. These studies, however, did not display the full force of the dual effects since the control period is too short for any extra learning effort to be paid off in the future. To fully appreciate the interplay between learning and control, a N-period control problem ( $N \gg 3$ ) must be considered. Unfortunately, the optimal dual control solution cannot be obtained numerically for  $N > 3$ . This motivates several recent developments of dual control methods by Tse *et al.* (refs. 8-10), Chow (ref. 11), MacRae (ref. 12), Alster and Belanger (ref. 13), and Alspach (ref. 14). The method developed by Tse *et al.* has been studied to a great extent and is applicable to a fairly large class of nonlinear stochastic systems, whereas the other methods are developed for specific cases of interests.

We shall discuss here the recent developments in dual control methods. The method proposed by Tse *et al.* (ref. 8) is discussed in detail, whereas the other methods are only described briefly due to their specific nature. Simulation studies are summarized and the future of dual control is discussed.

## II. DUAL CONTROL: AN ACTIVE CONTROLLER

To begin our discussions, let us first introduce the concept of dual control. Consider a linear static system given by

$$x_k = \alpha u_k + \epsilon_k ; \quad k = 1, 2 \quad (1)$$

where  $x_k$  is the observation,  $\alpha$  is the unknown parameter which is Gaussian with mean  $\alpha_0$  and variance  $\sigma_0$ ,  $u_k$  is the control, and  $\{\epsilon_k\}$  is a sequence of equidistributed and independent Gaussian variables with mean zero and variance  $q$ . The stochastic control problem is to find a control law

$$u_1 = u_1(q, \alpha_0, \sigma_0) : \quad u_2 = u_2(x_1, u_1, q, \alpha_0, \sigma_0)$$

such that the cost

$$J = E\left\{x_1 + x_2 + \frac{1}{2} u_1^2 + \frac{1}{2} u_2^2\right\} \quad (2)$$

is minimized. First, note that since equation (1) is a static linear system, if  $\alpha$  is known, the optimal closed-loop law would be

$$u_1^* = u_2^* = -\alpha \quad (3)$$

For unknown  $\alpha$ , the certainty equivalence (CE) controller is

$$u_1^{CE} = -\alpha_0 ; \quad u_2^{CE} = -\alpha_1 \quad (4)$$

where  $\alpha_1$  is the *updated* mean of  $\alpha$  after  $(x_1, u_1^{CE})$  are being observed.



Let  $u_1$  be an arbitrary control value applied to equation (1). After the observation of  $x_1$ , one can compute the *a posteriori* distribution for  $\alpha$  which is Gaussian with mean

$$\alpha_1 = \frac{\alpha_0 q + \sigma_0 u_1 x_1}{u_1^2 \sigma_0 + q} \quad (5)$$

and variance

$$\sigma_1 = \frac{\sigma_0 q}{\sigma_0 u_1^2 + q} \quad (6)$$

From equation (6), note that, if  $|u_1|$  is large, the updated variance for  $\alpha$  will be small; in fact, if  $|u_1| \rightarrow \infty, \sigma_1 \rightarrow 0$ . This implies that a large control value at  $k = 1$  helps learning of the unknown parameter  $\alpha$ . On the other hand, if  $u_1 \approx 0, \alpha_1 \approx \alpha_0$  and  $\sigma_1 \approx \sigma_0$ , that is, no additional knowledge on  $\alpha$  will be provided if a small control value is used at  $k = 1$ . To see what should be the proper or optimum value for  $u_1$ , one has to calculate the *value, or the usefulness, of additional information on  $\alpha$* , in the context of the given control problem, so that one can trade off present control (set  $u_1 = -\alpha_0$ ) with additional learning for future control (set  $|u_1|$  large).

Let  $u_1$  be the value of the control at  $k = 1$  and  $x_1$  be the resulting (random) observation. In minimizing the remaining cost to go,

$$J_2 = E\left\{x_2 + \frac{1}{2} u_2^2 | x_1, u_1\right\} = E\{\alpha u_2 | x_1, u_1\} + \frac{1}{2} u_2^2 = \alpha_1 u_2 + \frac{1}{2} u_2^2 \quad (7)$$

where  $\alpha_1$  is given by equation (5), the optimum control law at  $k = 2$  is

$$u_2 = -\alpha_1 = -\frac{\alpha_0 q + \sigma_0 u_1 x_1}{u_1^2 \sigma_0 + q} \quad (8)$$

Note that  $u_2$  is random since  $x_1$  is random. The minimum cost to go:

$$J_2^*(x_1, u_1) = -\frac{1}{2} \alpha_1^2 = -\frac{1}{2} \left( \frac{\alpha_0 q + \sigma_0 u_1 x_1}{u_1^2 \sigma_0 + q} \right)^2 \quad (9)$$

is a random variable through  $x_1$ . The cost of learning is negative and is represented by the expected minimum cost to go:

$$\begin{aligned}
E\{J_2^*(x_1, u_1)\} &= -\frac{1}{2} E \left( \frac{\alpha_0 q + \sigma_0 u_1 x_1}{u_1^2 \sigma_0 + q} \right)^2 \\
&= -\frac{1}{2} \left\{ \alpha^2 + \frac{\sigma_0^2 u_1^2}{u_1^2 \sigma_0 + q} \right\}
\end{aligned} \tag{10}$$

This indicates that learning is beneficial (negative cost) to future control. The overall cost of applying  $u_1$  is

$$\begin{aligned}
J &= E\left\{x_1 + \frac{1}{2} u_1^2\right\} + E\{J_2^*(x_1, u_1)\} \\
&= \alpha_0 u_1 + \frac{1}{2} u_1^2 - \frac{1}{2} \left\{ \alpha^2 + \frac{\sigma_0^2 u_1^2}{u_1^2 \sigma_0 + q} \right\}
\end{aligned} \tag{11}$$

The optimum value for  $u_1$  that will balance between present control (minimizing  $\alpha_0 u_1 + (1/2)u_1^2$ ) and additional learning for future control (minimizing  $-(1/2)\{\alpha_0^2 + (\sigma_0^2 u_1^2)/(u_1^2 \sigma_0 + q)\}$ ) is obtained by minimizing the "dual cost" given by equation (11).

The two components of the cost

$$c_1(u) = \alpha_0 u + \frac{1}{2} u^2 \text{ (direct control cost)} \tag{12}$$

$$c_2(u) = -\frac{1}{2} \left\{ \alpha_0^2 + \frac{\sigma_0^2 u^2}{u^2 \sigma_0 + q} \right\} \text{ (indirect control cost)} \tag{13}$$

are plotted in figure 1 (where  $\alpha_0$  is assumed positive). From the figure, it is clear that

(1) if  $q$  is small,  $u_1^* \approx -\alpha_0$ , since, in this case, unplanned (or accidental) learning is sufficient to reduce the updated variance (eq. (6)).

(2) if  $q$  is moderate,  $u_1^*$  is to the left of  $-\alpha_0$ , indicating that planned learning does pay off.

(3) if  $q$  is very large,  $u_1^* \approx -\alpha_0$ , since, in the case of high noise intensity, additional learning *does not* pay off and thus the controller might as well look at it as a one-stage optimization problem.

Similar conclusions can be drawn for different range of values for  $\sigma_0$ . In fact, the optimum  $u_1$  is determined by the ratio  $q/\sigma_0$  and  $\sigma_0$ .

In this static example, the effect of the control value does not propagate in time through the system, and thus the effects of learning and control can be nicely distinguished. For a dynamic system, control applied at a certain time will influence the future state trajectory, and thus the *dual effects* of the control would be difficult to separate. This example serves to point out one very important concept of dual control, namely, that planned learning should be done only if (1) accidental learning is not adequate or (2) planned learning does pay off. This is the central theme of dual control theory.

### III. PROBLEM FORMULATION AND OPTIMAL SOLUTION

In this section, the formulation and solution of the optimal stochastic control problem for discrete-time systems are discussed. Consider a discrete-time nonlinear system described by

$$\underline{x}_{k+1} = \underline{f}_k(\underline{x}_k, \underline{u}_k) + \underline{v}_k ; \quad k = 0, 1, \dots, N-1 \quad (14)$$

and nonlinear observation described by

$$\underline{y}_k = \underline{h}_k(\underline{x}_k) + \underline{w}_k ; \quad k = 1, \dots, N \quad (15)$$

where  $\underline{x}_0$  is the initial condition, a random variable with mean  $\hat{\underline{x}}_0|_0$  and covariance  $\Sigma_0|_0$ ;  $\{\underline{v}_k\}$  and  $\{\underline{w}_k\}$  are the sequences of process and measurement noises, respectively, mutually independent, white and with known statistics. For simplicity, they may be assumed to have zero mean. Consider further the performance measure

$$J = E \left\{ K(\underline{x}_N) + \sum_{k=0}^{N-1} L_k(\underline{x}_k, \underline{u}_k) \right\} \quad (16)$$

where the expectation  $E\{\cdot\}$  is taken over all underlying random quantities. Finally, consider admissible control of the closed-loop type:

$$\underline{u}_k = \underline{u}_k(\underline{y}^k, \underline{U}^{k-1}), \underline{y}^k \triangleq \{\underline{y}_1, \dots, \underline{y}_k\}, \underline{U}^{k-1} \triangleq \{\underline{u}_0, \dots, \underline{u}_{k-1}\} \quad (17)$$

The goal is to find the optimal control law  $\{\underline{u}_k^*\}_{k=0}^{N-1}$  which has the form of equation (17) and minimizes the cost (eq. (16)) subject to the constraints (eqs. (14) and (15)).

Suppose that a certain control law  $\underline{u}_k(\cdot)$  is chosen for  $k = 0, 1, \dots, N-2$ , and the observation sequence  $\underline{y}^{N-1}$  is observed. The remaining problem is to select  $\underline{u}_{N-1}$  so that the conditional expected cost

$$E\{J|Y^{N-1}\} = E\{J|Y^{N-1}, \bar{U}^{N-2}\}$$

$$= \sum_{k=0}^{N-2} E\{L_k(\underline{x}_k, \bar{u}_k) | Y^{N-1}, \bar{U}^{N-2}\} + E\{L_{N-1}(\underline{x}_{N-1}, \underline{u}_{N-1}) + K(\underline{x}_N) | Y^{N-1}, \bar{U}^{N-2}\} \quad (18)^1$$

is minimized. Since the summation term is independent of  $\underline{u}_{N-1}$ , the optimum  $\underline{u}_{N-1}$  is obtained by minimizing

$$\begin{aligned} & E\{L_{N-1}(\underline{x}_{N-1}, \underline{u}_{N-1}) + K(\underline{x}_N) | Y^{N-1}, \bar{U}^{N-2}\} \\ &= \int \{L_{N-1}(\underline{x}_{N-1}, \underline{u}_{N-1}) + K[f_{N-1}(\underline{x}_{N-1}, \underline{u}_{N-1}) \\ &\quad + \underline{v}_{N-1}]\} p(\underline{x}_{N-1}, \underline{v}_{N-1} | Y^{N-1}, \bar{U}^{N-2}) d\underline{x}_{N-1} d\underline{v}_{N-1} \\ &= \int \{L_{N-1}(\underline{x}_{N-1}, \underline{u}_{N-1}) + K[f_{N-1}(\underline{x}_{N-1}, \underline{u}_{N-1}) \\ &\quad + \underline{v}_{N-1}]\} p(\underline{x}_{N-1} | Y^{N-1}, \bar{U}^{N-2}) p(\underline{v}_{N-1}) d\underline{x}_{N-1} d\underline{v}_{N-1} \end{aligned} \quad (19)$$

Note that  $p(\underline{v}_{N-1})$  is known *a priori*, and thus the only on-line quantity needed in the calculation of equation (19) is either  $(Y^{N-1}, \bar{U}^{N-2})$  or  $p(\underline{x}_{N-1} | Y^{N-1}, \bar{U}^{N-2})$ ; thus the optimal control must be of the form

$$\underline{u}_{N-1}^* = \underline{u}_{N-1}^*(Y^{N-1}, \bar{U}^{N-2}) \quad \text{or} \quad \underline{u}_{N-1}^* = \underline{u}_{N-1}^*[p(\underline{x}_{N-1} | Y^{N-1}, \bar{U}^{N-2})] \quad (20)$$

Substitute equation (20) into (19) and define the optimal cost to go:

$$I_{N-1}(\psi_{N-1}) = E\{L_{N-1}[\underline{x}_{N-1}, \underline{u}_{N-1}^*(\psi_{N-1})] + K\{f_{N-1}[\underline{x}_{N-1}, \underline{u}_{N-1}^*(\psi_{N-1})] + \underline{v}_{N-1}\} | \psi_{N-1}\} \quad (21)$$

where  $\psi_{N-1}$  is the *information state* that can either be represented by  $(Y^{N-1}, \bar{U}^{N-2})$  or  $p(\underline{x}_{N-1} | Y^{N-1}, \bar{U}^{N-2})$ . The cost of using the control law

$\{\underline{u}_k(\cdot)\}_{k=0}^{N-2}$ , with the knowledge that the remaining control law is optimal, is

$$\sum_{k=0}^{N-2} E\{L_k(\underline{x}_k, \bar{u}_k) | \psi_{N-1}\} + I_{N-1}(\psi_{N-1}) \quad (22)$$

<sup>1</sup>When a control law is specified, notation  $\bar{u}_i$  is used to denote the realization of the control value at step  $i$ ;  $\bar{U}^k \triangleq (\bar{u}_0, \dots, \bar{u}_k)$  when  $Y^k$  is realized.

To carry this procedure backward in time, assume that  $\underline{u}_k(\cdot)$  is chosen,  $k = 0, 1, \dots, N-3$ , and  $\underline{Y}^{N-2}$  is observed. If a particular control value  $\underline{u}_{N-2}$  is chosen, then, from equations (14) and (15), a dynamic description of the information state is available:

$$\psi_{N-1} = \psi_{N-1}(\psi_{N-2}, \underline{u}_{N-2}, \underline{Y}_{N-1}) \quad (23)$$

where  $\psi_{N-2}$  is the information state at  $N-2$  and  $\underline{Y}_{N-1}$  is the random observation to be observed at time  $N-1$ . If, for example, the conditional density for the current state is used as an information state, then

$$\begin{aligned} \psi_{N-1} &= p(\underline{x}_{N-1} | \underline{Y}^{N-1}, \bar{\underline{U}}^{N-3}, \underline{u}_{N-2}) = \frac{1}{C_k} p(\underline{y}_{N-1} | \underline{x}_{N-1}) \int p(\underline{x}_{N-1} | \underline{x}_{N-2}, \underline{u}_{N-2}) \times \\ &\quad p(\underline{x}_{N-2} | \underline{Y}^{N-2}, \bar{\underline{U}}^{N-3}) d\underline{x}_{N-2} \quad \left. \vphantom{\int} \right\} \quad (24) \\ &= \frac{1}{C_k} p(\underline{y}_{N-1} | \underline{x}_{N-1}) \int p(\underline{x}_{N-1} | \underline{x}_{N-2}, \underline{u}_{N-2}) \psi_{N-2} d\underline{x}_{N-2} \end{aligned}$$

The optimal cost to go is given by

$$I_{N-2}(\psi_{N-2}) = \min_{\underline{u}_{N-2}} E \{ L_{N-2}(\underline{x}_{N-2}, \underline{u}_{N-2}) + I_{N-1}[\psi_{N-1}(\psi_{N-2}, \underline{u}_{N-2}, \underline{Y}_{N-1})] | \psi_{N-2} \} \quad (25)$$

and the cost of using the control law  $\{\underline{u}_k(\cdot)\}_{k=0}^{N-3}$ , with the knowledge that the remaining control law is optimal, is

$$\sum_{k=0}^{N-3} E\{L_k(\underline{x}_k, \bar{\underline{u}}_k) | \psi_{N-2}\} + I_{N-2}(\psi_{N-2}) \quad (26)$$

Inductively, the conditional expected optimal cost to go,  $I_k(\psi_k)$ , satisfies the dynamic programming equation:

$$I_k(\psi_k) = \min_{\underline{u}_k} E\{L_k(\underline{x}_k, \underline{u}_k) + I_{k+1}[\psi_{k+1}(\psi_k, \underline{u}_k, \underline{Y}_{k+1})] | \psi_k\} \quad (27)$$

with end condition

$$I_N(\psi_N) = E\{K(\underline{x}_N) | \psi_N\} \quad (28)$$

The optimal control law is obtained by solving equations (27) and (28).

It is well known that the solution of equations (27) and (28) is feasible only for a small class of problems (refs. 3 and 8). In most cases, one is

unable to solve equations (27) and (28) due to the curse of dimensionality (ref. 2), and thus suboptimal procedures must be developed for the problem. However, one can deduce the dual nature of the optimal control from the dynamic programming equation (27). Depending on the actual realization of  $\psi_k$ , the overall cost of applying the control  $\underline{u}_k$  at time  $k$  consists of two components: direct cost,  $E\{L_k(\underline{x}_k, \underline{u}_k) | \psi_k\}$ , and indirect cost,  $E\{I_{k+1}[\psi_{k+1}(\psi_k, \underline{u}_k, \underline{y}_{k+1})] | \psi_k\}$ . The direct cost represents the conditional expected cost that would have been incurred if the problem is a one-shot static problem, whereas the indirect cost represents the conditional expected cost of the future information state, which depends on  $\underline{u}_k$  because of the dynamic description of the system equation (14) and the observation relation equation (15). The dual effects of  $\underline{u}_k$  are interrelated and embedded in the indirect cost component.

#### IV. A DUAL CONTROL METHOD

In this section, we shall describe a dual control method due to Tse *et al.* (refs. 8 and 9). Note that the main difficulty in solving equation (27) hinges on the calculation of the indirect cost  $E\{I_{k+1}[\psi_{k+1}(\psi_k, \underline{u}_k, \underline{y}_{k+1})] | \psi_k\}$ . If, however, this quantity can be obtained exactly or approximately, then at every time  $k$ , it looks like we have a two-stage stochastic control problem. With this motivation, Tse *et al.* propose to approximate equation (27) as follows:

(1) Approximate the information state  $\psi_k$  so that the dimension of the information state space stays constant in time. It is suggested that the conditional mean and covariance be used as the approximated information state. Denote such an approximated information state by  $\hat{\psi}_k = \{\hat{\underline{x}}_k | k, \underline{\Sigma}_k | k\}$ .<sup>2</sup>

(2) Approximate the optimal cost-to-go function associated with an approximated information state by performing a second-order perturbation analysis around a nominal trajectory that is *different for different predicted approximated information states*. Denote the approximation for the optimal to go by  $\hat{I}_{k+1}[\hat{\psi}_{k+1}(\hat{\psi}_k, \underline{u}_k, \underline{y}_{k+1})]$ , which is given by (see refs. 8 and 15 for details)

$$\hat{I}_{k+1}[\hat{\psi}_{k+1}(\hat{\psi}_k, \underline{u}_k, \underline{y}_{k+1})] = J_D(N - k) + J_C(N - k) + J_P(N - k) \quad (29)$$

where

$$J_D(N - k) = K(\underline{x}_N^0) + \sum_{j=k+1}^{N-1} L_j(\underline{x}_j^0, \underline{u}_j^0) + \gamma_0(k+1) + L_k(\hat{\underline{x}}_k | k + \underline{u}_k) \quad (30)$$

is the deterministic part of the cost along the nominal

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<sup>2</sup>We denote the conditional error covariance by either  $\underline{\Sigma}_k | k$  or  $\underline{\Sigma}(k | k)$ .

$$\underline{x}_{j+1}^0 = f_j(\underline{x}_j^0, \underline{u}_j^0), j = k+1, \dots, N-1; \underline{x}_{k+1}^0 = E\{\underline{x}_{k+1} | \psi_k, \underline{u}_k\}$$

and

$$J_C(N-k) \triangleq \frac{1}{2} \text{tr}[\underline{K}_0(k+1)\underline{\Sigma}(k+1|k)] + \frac{1}{2} \sum_{j=k+1}^{N-1} \text{tr}[\underline{K}_0(j+1)\underline{Q}(j)] \quad (31)$$

$$J_P(N-k) \triangleq \frac{1}{2} \sum_{j=k+1}^{N-1} \text{tr}[A_{o,\underline{xx}}(j)\underline{\Sigma}_0(j|j)] \quad (32)$$

are the stochastic terms in the cost evaluated along the nominal. The matrices  $\underline{K}_0(j)$ ,  $A_{o,\underline{xx}}(j)$  and the scalar  $\gamma_0(j)$  are given by

$$\left. \begin{aligned} \gamma_0(j) &= \gamma_0(j+1) - \frac{1}{2} H'_{o,\underline{u}}(j) \bar{\mathcal{H}}_{o,\underline{uu}}^{-1}(j) H_{o,\underline{u}}(j) \\ j &= N-1, \dots, k+1; \gamma_0(N) = 0 \end{aligned} \right\} \quad (33)$$

$$\left. \begin{aligned} \underline{P}_0(j) &= H_{o,\underline{x}}(j) - \mathcal{H}'_{o,\underline{ux}}(j) \bar{\mathcal{H}}_{o,\underline{uu}}^{-1}(j) H_{o,\underline{u}}(j) \\ j &= N-1, \dots, k+1; \underline{P}_0(N) = \psi_{o,\underline{x}} \end{aligned} \right\} \quad (34)$$

$$\left. \begin{aligned} \underline{K}_0(j) &= \mathcal{H}_{o,\underline{xx}}(j) - A_{o,\underline{xx}}(j) \\ j &= N-1, \dots, k+1; \underline{K}_0(N) = \psi_{o,\underline{xx}} \end{aligned} \right\} \quad (35)$$

with

$$H_0(j) \triangleq L_0(j) + \underline{p}'_0(j+1) \underline{f}_0(j) \quad (36)^3$$

$$\mathcal{H}_{o,\underline{xx}}(j) \triangleq H_{o,\underline{xx}}(j) + \underline{f}'_{o,\underline{x}}(j) \underline{K}_0(j+1) \underline{f}_{o,\underline{x}}(j) \quad (37)$$

$$\mathcal{H}_{o,\underline{ux}}(j) = H_{o,\underline{ux}}(j) + \underline{f}'_{o,\underline{u}}(j) \underline{K}_0(j+1) \underline{f}_{o,\underline{x}}(j) \quad (38)$$

$$\mathcal{H}_{o,\underline{uu}}(j) = H_{o,\underline{uu}}(j) + \underline{f}'_{o,\underline{u}}(j) \underline{K}_0(j+1) \underline{f}_{o,\underline{u}}(j) \quad (39)$$

$$A_{o,\underline{xx}}(j) = \mathcal{H}_{o,\underline{ux}}(j) \mathcal{H}_{o,\underline{uu}}^{-1}(j) \mathcal{H}_{o,\underline{ux}}(j) \quad (40)$$

<sup>3</sup>  $L_0(j) \triangleq L_j(\underline{x}_j^0, \underline{u}_j^0)$ ,  $\underline{f}_0(j) \triangleq \underline{f}_j(\underline{x}_j^0, \underline{u}_j^0)$ .

(3) At each time  $k$ , an on-line optimization problem

$$\min_{\underline{u}_k} E\{L_k(\underline{x}_k, \underline{u}_k) + \hat{I}_{k+1}[\hat{\psi}_{k+1}(\hat{\psi}_k, \underline{u}_k, \underline{y}_{k+1})] | \hat{\psi}\} \quad (41)$$

is solved in real time. Denote the minimizing  $\underline{u}_k$  by  $\underline{u}_k^{DC}(\hat{\psi}_k)$ . Apply  $\underline{u}_k^{DC}(\hat{\psi}_k)$ , update  $\hat{\psi}_{k+1}$  when  $\underline{y}_{k+1}$  is observed, and resolve the problem for  $k + 1$ .

One distinguishing characteristic of this approach is that it explicitly takes into account that future measurements will be made, and these future measurements will provide additional information about the uncertain parameters that will enhance future control. Therefore, learning is planned in accordance with the control objective. This gives the control law a closed-loop characteristic (see refs. 15 and 16). To see this, first note that  $\gamma_0(k+1)$ ,  $p(k+1)$ , and  $K_0(k+1)$  are *not* stochastic and depend only on the nominal control sequence being chosen. The function  $H_0(j)$  defined by equation (36) is the Hamiltonian for the corresponding deterministic optimal control problem. Therefore, if the nominal control is a local minimum for the deterministic problem, then  $H_{0,\underline{u}}(j) = 0$  and thus  $\gamma_0(j) = 0$ ; also  $\mathcal{H}_{0,\underline{u}\underline{u}}(j)$  is positive definite for all  $j \geq k+1$ . The latter implies that the perturbation analysis is valid. In fact, a necessary and sufficient condition for the existence and uniqueness of the optimum perturbation solution is that  $\mathcal{H}_{0,\underline{u}\underline{u}}(j)$  must be positive definite (regardless of whether the nominal control is a local minimum). If  $\mathcal{H}_{0,\underline{u}\underline{u}}(j)$  is positive definite, then  $A_{0,\underline{x}\underline{x}}(j)$  (defined by eq. (40)) must be nonnegative definite, with rank equal to the dimension of the control vector. Now observe that the first stochastic term (eq. (31)) reflects the effect of the uncertainty at time  $k$  and subsequent process noises on the cost. These uncertainties cannot be affected by  $\underline{u}(k)$ , but their weightings depend on it. The effect of these uncontrollable uncertainties on the cost should be minimized by the control; this term indicates the need for the control to be cautious and thus is called the *caution term*. The second stochastic term, equation (32), accounts for the effect of uncertainties when subsequent decisions will be made. As discussed above, if the perturbation problem has a solution, then the weighting of these future uncertainties is nonnegative ( $A_{0,\underline{x}\underline{x}}$  is positive semidefinite). If the control can reduce by probing (experimentation) the future updated covariances, it can thus reduce the cost. The weighting matrix  $A_{0,\underline{x}\underline{x}}$  yields approximately the value of future information for the problem under consideration. Therefore, this is called the *probing term*. Since the rank of  $A_{0,\underline{x}\underline{x}}(j)$  is equal to the dimension of the control vector, this indicates that, along a certain nominal sequence, certain combinations of state uncertainties do not contribute (up to second order) to the total average cost.

The benefit of probing is weighted by its cost and a compromise is chosen to minimize the sum of the deterministic, caution, and probing terms. The minimization will also achieve a tradeoff between the present and future actions according to the information available when the corresponding decisions are made.



Figure 2 is a flow chart of the algorithm.

## V. OTHER DUAL CONTROL METHODS

Several other dual control methods are suggested in the literature. But almost all are developed for limited classes of problems. Chow (ref. 11) and MacRae (ref. 12) developed two different dual control methods mainly for the control of linear econometric models with unknown parameters. Chow's method is based on an approximation of the optimal cost to go. In this respect, his algorithm is similar to the one described in section IV. There are some basic differences between Chow's method and the one developed by Tse *et al.* One is that Chow employed a single nominal whereas Tse *et al.* allowed to have multi-nominals; another difference is that Chow proposed a step-by-step numerical approximation of the optimal cost to go, whereas Tse *et al.* used perturbation analysis to obtain the optimal cost to go. MacRae's approach is also based on dynamic programming, but whenever a future updated covariance appeared in the equation, she replaced it by a first-order approximation about a nominal trajectory.

Alster and Belanger (ref. 13) developed an ad-hoc dual control method for the control of a class of linear discrete time systems with unknown constant parameters. They first rewrite the expected cost conditioned to the current time period, and suggested to minimize the open-loop expected cost subject to an added constraint that the next period's updated covariance is smaller than some threshold. While this is a plausible approach to the problem, the problem of selecting the threshold is nontrivial and probably highly problem dependent. Kalaba and Detchmندی (ref. 17) suggested a similar approach where the original cost is modified to include penalty of having bad estimates. Alspach (ref. 14) used perturbation methods to develop a dual control method for the control of linear systems with *known* dynamics but with multiplicative noise.

All methods described in this section assumed a quadratic criterion.

## VI. SIMULATION STUDIES

Up until now, no simulation study using the methods proposed by Chow, MacRae, and Kalaba and Detchmندی has been reported in the literature. Limited simulation studies were reported in reference 13 where their algorithm is compared to the certainty equivalence (CE) control algorithm for the control of single-input/single-output linear systems with unknown parameters under the minimum output variance criterion. Alspach (ref. 14) also compared his algorithm with the CE algorithm for the control of single-input/single-output linear systems with multiplicative noise under the quadratic criterion. In both studies, the proposed dual control methods perform better than the CE method.

Tse *et al.* applied their dual control methods to several representative cases in three different classes of problems. In each case, the dual control method is an order of magnitude better than the CE method. Moreover, the simulation studies revealed several important characteristics of dual control. The first class of problems is the control of linear systems with unknown parameters. Tse *et al.* (refs. 8 and 9) studied the control of single-input/single-output linear systems with unknown parameters under the quadratic criterion. By varying the weighting matrices, the active learning characteristics of the dual control is vividly displayed. The second class of problems is that of controlling nonlinear systems. For such a class of problems, there is no unknown parameter to be learned. However, there is another important characteristic of the dual control: the capability of bringing the system state to a certain region that will improve state estimation. This is clearly shown in the studies by Tse and Bar-Shalom (ref. 10), where they applied their dual control method to an interception problem. The third class is the control of dynamic systems with drifting parameters. Pekelman and Tse (ref. 18) used the dual control method in the problem of optimum decision on advertisement expenditure. In this case, the system is nonlinear with drifting (unknown) parameters. This study also indicates that when the knowledge of the drifting parameters is crucial to the control performance, the dual control method will induce enough learning to track the variation of the parameters.

## VII. CONCLUDING REMARKS AND THE FUTURE OF DUAL CONTROL THEORY

Generally, it is found that dual control methods give a better performance, but they are also considerably more complicated than the CE control method. The question then is how much we want to trade off between performance improvement and complexity. In some cases, it can be shown that the CE method gives reasonably good results (ref. 19) or even the optimal solution (refs. 16, 20, and 21), and thus the dual control is not needed. Clearly, one important question to be answered in any stochastic control problem is how much we can expect to gain by using dual control instead of CE control. Theoretical studies in this direction are not found in the literature. From personal experience, a problem with the following characteristics may need dual control approach:

(1) The control period is relatively short, but not too short, so that the benefit of learning does not contribute substantially to the indirect cost.

(2) State estimation performance is highly dependent on the state trajectory.

(3) Control performance is highly dependent on the knowledge of the unknown parameters in the system.

(4) The natural tendency of the system is incompatible with the system performance, for example, regulating the output of an unstable system close to zero or steering the output of a stable system to a specified value.

To understand the similarities and differences of all the proposed dual control methods, comparison studies of these methods should be carried out using the same data base. Bar-Shalom and Tse (ref. 22) initiated effort in this direction by comparing the methods of Tse *et al.* (ref. 8), Chow (ref. 11), and MacRae (ref. 12) via a simple three-stage linear system with unknown feed-forward gain parameter. Comparison studies for more complicated problems will be reported in the future.

Throughout the discussions, it was assumed that the structural form of the system dynamic and the observation relation are known to the controller. While this assumption is valid in many engineering applications, it may not be valid in many processes arising in economic, chemical, and biological systems where a unique description of the system structure is impossible. A formulation of combined system identification and control for systems with ambiguity in structural description seems to be needed.

One possible formulation is to assume that the underlying system can have  $M$  different possible structural descriptions. These descriptions may be derived from several equally possible hypothetical assumptions. The problem then is that of minimizing a certain cost criterion, subject to the constraint that the *true* dynamic equation can be one of the  $M$  possible difference equations. This problem has been considered in references 23 through 26. These approaches are equivalent to separation of detection and control. While the methods seem to be useful in some applications (ref. 27), in some cases, the method faces the problem of identifiability where the "true" system cannot be identified even at  $k \rightarrow \infty$ . (For a discussion of identifiability problems, see refs. 28 to 32.) An active approach similar to the dual control (as described in section VI) would be useful. In this case, the control would have the following "dual" purposes.

- (1) To allow resolvability of model structures and to help in estimating the unknown parameters of a given model structure.

- (2) To regulate or control the system to follow some desired path or performance measure.

For  $M$  different possible model structure descriptions, the problem should be formulated as a simultaneous hypothesis testing and control problem. Such an active approach has not yet been attempted.

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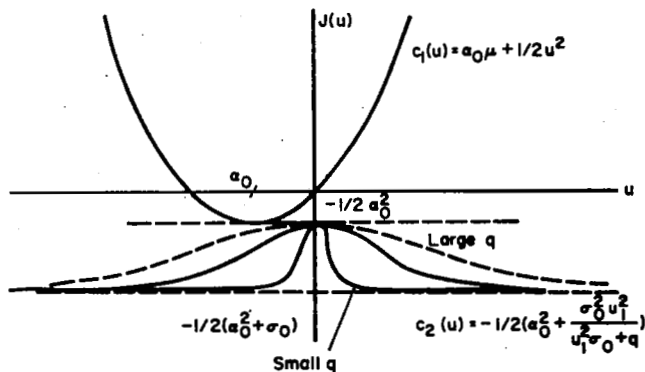


Figure 1.- Decomposition of dual cost.

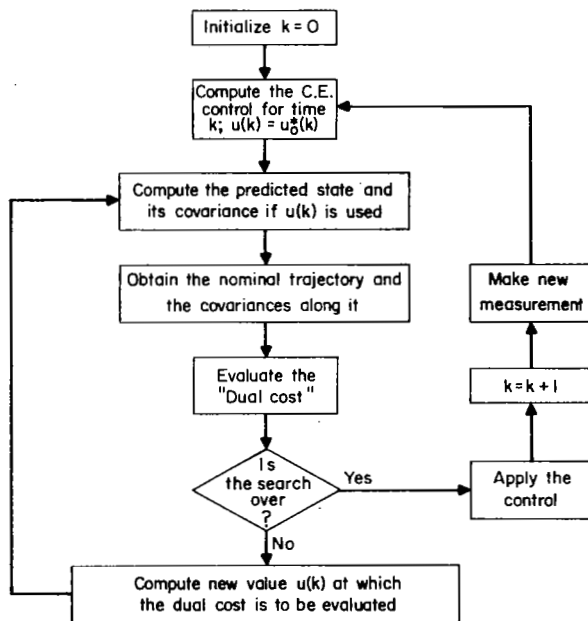


Figure 2.- Flow chart of dual control algorithm.





# CLOSED-LOOP STRUCTURAL STABILITY FOR LINEAR-QUADRATIC OPTIMAL SYSTEMS\*

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## SUMMARY

This paper contains an explicit parametrization of a subclass of linear constant gain feedback maps that never destabilize an originally open-loop stable system. These results can then be used to obtain several new structural stability results for multiinput linear-quadratic feedback optimal designs.

## I. INTRODUCTION AND MOTIVATION

This paper presents preliminary results which, in our opinion, represent a first necessary step in the systematic computer-aided design of reliable control systems for future aircraft. It is widely recognized that advances in active control aircraft and control configured vehicles will require the automatic control of several actuators so that future aircraft can be flown characterized by reduced stability margins and additional flexure modes.

As a starting point for our motivation, we must postulate that the design of future stability augmentation systems must be a multivariable design problem. As such, traditional single-input/single-output system design tools based on classical control theory cannot be effectively used, especially in a computer-aided design context. Since modern control theory provides a conceptual theoretical and algorithmic tool for design, especially in the linear-quadratic-Gaussian (LQG) context (see, e.g., ref. 1), it deserves a special look as a starting point in the investigation.

In spite of the tremendous explosion of reported results in LQG multivariable design, the robustness properties have been neglected. Experience has shown that LQG designs "work" very well if the mathematical models upon which the design is based are somewhat accurate. There are several sensitivity studies involving "small-parameter perturbations" associated with the LQG problem. We submit, however, that the general problem of sensitivity and even stability of multivariable LQG designs under *large parametric and structural* changes is an open research area.

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It is useful to reflect on the basic methodology in classical servomechanism theory which dealt with such large-parameter changes. The overall sensitivity and stability considerations were captured in the definition of *gain* and *phase margins*. If a closed-loop system were characterized by reasonable gain and phase margins, then (a) reasonable changes in the parameters of the open-loop transfer functions and (b) changes in the loop gains due, for example, to saturation and other nonlinearities could be accommodated with guaranteed stability and at the price of somewhat degraded performance.

Although LQG designs are time-domain oriented, nonetheless their frequency-domain interpretations are important, although not universally appreciated. For example, for the single-input/single-output linear-quadratic (LQ) optimal designs, Anderson and Moore (ref. 2) have shown that LQ-optimal designs are characterized by (a) an infinite gain margin property and (b) a phase margin of at least  $60^\circ$ . Such results are valuable because it can be readily appreciated that, at least in the single-input/single-output case, modern control theory designs tend to have a good degree of robustness, as measured by the classical criteria of gain and phase margin.

Advances in the multiinput/multioutput case, however, have been scattered and certainly have not arrived at the cookbook design stage. Multivariable system design is extremely complex.<sup>1</sup> To a certain extent, the *numerical solution* of LQ optimal is very easy. However, fundamental understanding of the structural interdependencies and their interactions with the weighting matrices is not a trivial matter. We believe that such fundamental understanding is crucial for robust designs as well as for reliable designs that involve a certain degree of redundancy in controls and sensors.

The recent S.M. thesis by Wong (ref. 3) represents a preliminary yet positive contribution in this area. In fact, the technical portion of this paper represents a slight modification of some of the results reported in reference 3. In particular, we focus our attention on the stability properties of closed-loop systems designed on the basis of LQ-optimal techniques when the system matrices and loop gains undergo large variations.

The main contributions reported here are the eventual results of generalizing the concepts of gain margin and of performing large-perturbation sensitivity analysis for multivariable linear systems designed via the LQ approach. We warn the reader that much additional theoretical and applied research is needed before the implications of these theoretical results can (a) be fully understood and (b) translated into systematic "cookbook" procedures that have the same value as the conventional results in classical servomechanism design.

This paper is organized as follows: in section 2 we present an explicit parametrization of a subclass of linear constant feedback maps that never destabilize an originally open-loop stable system and establish some of its properties. In section 3, we apply this construct to obtain several new

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<sup>1</sup>Even the notion of what constitutes a "zero" of a multivariable transfer matrix was not fully appreciated until recently.

closed-loop structural stability characterizations of multiinput LQ-optimal feedback maps. We conclude in section 4 with a brief discussion of the relevance of the present results for computer-aided iterative feedback design.

### Notation

- (1) The linear time-invariant system

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}(t) + \underline{B}u(t)$$

$$\underline{z}(t) = \underline{H}^T \underline{x}(t)$$

where

$$\underline{x}(t) \in R^n \quad \underline{x}(\cdot) = \text{state vector}$$

$$\underline{u}(t) \in R^m \quad \underline{u}(\cdot) = \text{control vector}$$

$$\underline{z}(t) \in R^r \quad \underline{z}(\cdot) = \text{output vector}$$

and  $\underline{A} \in R^{n \times n}$ ,  $\underline{B} \in R^{n \times m}$ , and  $\underline{H}^T \in R^{r \times n}$  will be denoted by  $\Sigma(\underline{A}, \underline{B}, \underline{H}^T)$ . Where  $\underline{H}^T$  is irrelevant to the discussion, we shorten the notation to  $\Sigma(\underline{A}, \underline{B})$  and, where the choice  $\underline{A}, \underline{B}$  is clear from the context, we simply use  $\Sigma$ .

If the matrix  $\underline{A}$  is stable (i.e., all eigenvalues of  $\underline{A}$  have strictly negative real parts), we refer to  $\Sigma(\underline{A}, \underline{B}, \underline{H}^T)$  as a stable system.

- (2)  $R(\underline{K})$  is range space of  $\underline{K}$ ,  $N(\underline{K})$  is null space (kernel) of  $\underline{K}$ , and  $\text{Rk}(\underline{K})$  is rank of  $\underline{K}$ .

- (3) Given system  $\Sigma(\underline{A}, \underline{B}, \underline{H}^T)$ ,

$$R(\underline{A}, \underline{B}) \triangleq \text{controllable subspace of the pair } (\underline{A}, \underline{B})$$

$$\triangleq R(\underline{B}) + \underline{A}R(\underline{B}) + \dots + \underline{A}^{n-1}R(\underline{B})$$

$$N(\underline{H}^T, \underline{A}) \triangleq \text{unobservable subspace of the pair } (\underline{H}^T, \underline{A})$$

$$\triangleq \bigcap_{i=1}^n N(\underline{H}^T \underline{A}^{i-1})$$

- (4) If  $\underline{Q} \in R^{n \times n}$  is positive semidefinite, we write  $\underline{Q} \geq \underline{0}$ . If  $\underline{Q}$  is positive definite, we write  $\underline{Q} > \underline{0}$ .

## II. PARAMETRIZATION OF NONDESTABILIZING FEEDBACK MAPS

We begin our discussion as follows.

### Definition 1

Given the stable system  $\Sigma(\underline{A}, \underline{B})$ , let

$$S(\Sigma) \triangleq \{ \underline{G}^T \in R^{m \times n} \mid (\underline{A} + \underline{B} \underline{G}^T) \text{ is stable} \}$$

that is,  $S(\Sigma)$  is the set of all feedback maps that never destabilize an originally open-loop stable system, where  $\underline{u}(t) \triangleq \underline{G}^T \underline{x}(t)$ .

Ideally, one would like to be able to explicitly parametrize  $S(\Sigma)$ , but as this is a well-known intractable problem, our strategy here is to look for a *simple* parametrization of a (hopefully) *sufficiently general subset* of  $S(\Sigma)$ .

We begin by first recalling some standard Lyapunov-type results:

Lemma 1 (Wonham, ref. 4):

(i) If  $\underline{A}$  is stable, then the Lyapunov equation

$$\underline{P} \underline{A} + \underline{A}^T \underline{P} + \underline{Q} = \underline{0}$$

with  $\underline{Q} \geq \underline{0}$  has a *unique* solution  $\underline{P} \geq \underline{0}$ . If, in addition,  $(\underline{Q}^{1/2}, \underline{A})$  is observable, then  $\underline{P} > \underline{0}$ .

(ii) If

$$(1) \quad \underline{P} \geq \underline{0}, \underline{Q} \geq \underline{0} \text{ satisfy } \underline{P} \underline{A} + \underline{A}^T \underline{P} + \underline{Q} = \underline{0}$$

$$(2) \quad (\underline{Q}^{1/2}, \underline{A}) \text{ is detectable}$$

Then  $\underline{A}$  is stable.

(iii) If  $\underline{Q} \geq \underline{0}$  and  $(\underline{Q}^{1/2}, \underline{A})$  is observable (detectable), then for all  $\underline{P} \geq \underline{0}$ ,  $\underline{R} > \underline{0}$  and for all  $\underline{B}, \underline{F}^T$ , the pair  $(\sqrt{\underline{Q}} + \underline{P} + \underline{F} \underline{R} \underline{F}^T, \underline{A} + \underline{B} \underline{F}^T)$  is observable (detectable).

Proof:

For (i), see reference 4, page 298.

For (ii), see reference 4, page 299.

For (iii), see reference 4, page 82.

To proceed, the following definition will be useful.

## Definition 2

For any stable  $\underline{A}$ , let

$$LP(\underline{A}) \triangleq \{\underline{K} \geq \underline{0} \mid \underline{K}\underline{A} + \underline{A}^T \underline{K} \leq \underline{0}\}$$

$$LP^+(\underline{A}) \triangleq \{\underline{K} > \underline{0} \mid \underline{K}\underline{A} + \underline{A}^T \underline{K} < \underline{0}\}$$

Remark:  $LP(\underline{A})$  is generally a proper subset of the set of all positive-semidefinite matrices of dimension  $n$ .

Example:

Suppose that

$$\underline{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad \lambda_1 < 0, \quad \lambda_2 < 0$$

Then

$$LP(\underline{A}) = \left\{ \begin{bmatrix} K_1 & K_{12} \\ K_{12} & K_2 \end{bmatrix} \mid \begin{array}{l} K_1 \geq 0 \\ K_2 \geq 0 \end{array}, K_1 K_2 \geq \left[ \frac{(\lambda_1 + \lambda_2)^2}{4\lambda_1 \lambda_2} \right] K_{12}^2 \right\}$$

Note that  $\begin{bmatrix} K_1 & K_{12} \\ K_{12} & K_2 \end{bmatrix} \geq \underline{0}$  iff  $K_1 \geq 0, K_2 \geq 0, K_1 K_2 \geq K_{12}^2$  and that

$$\frac{(\lambda_1 + \lambda_2)^2}{4\lambda_1 \lambda_2} \geq 1, \text{ with equality iff } \lambda_1 = \lambda_2.$$

Lemma 2:

- (i)  $LP(\underline{A})$  is a convex cone; that is,  $\underline{K}_1, \underline{K}_2 \in LP(\underline{A})$  implies  $\alpha_1 \underline{K}_1 + \alpha_2 \underline{K}_2 \in LP(\underline{A})$  for all  $\alpha_1 \geq 0, \alpha_2 \geq 0$ .
- (ii)  $\underline{K} \in LP(\underline{A}_1) \cap LP(\underline{A}_2)$  implies  $\underline{K} \in LP(\underline{A}_1 + \underline{A}_2)$ .
- (iii)  $\underline{K} \in LP(\underline{A})$  implies  $\underline{K} \in LP(\underline{A} + \underline{B}(\underline{S} - \underline{L})\underline{B}^T \underline{K})$ .

Proof:

Straightforward. We are now ready to introduce our first crucial result.

Lemma 3:

Let  $\underline{A}$  be stable.

Then  $[\underline{A} + (\underline{N} - \underline{M})\underline{K}]$  is stable for all  $\underline{K} \in LP(\underline{A})$  and for all  $\underline{M} \geq \underline{0}$ ,  $\underline{N} = -\underline{N}^T$  and where  $R(\underline{N}) \subset R(\underline{M})$ .

If  $\underline{K} \in LP^+(\underline{A})$ , then the condition  $R(\underline{N}) \subset R(\underline{M})$  can be omitted.

Proof:

Let  $\underline{Q} \triangleq -(\underline{K}\underline{A} + \underline{A}^T\underline{K})$ . Since  $\underline{K} \in LP(\underline{A})$ , we have  $\underline{Q} \geq \underline{0}$ , and  $\underline{A}$  stable implies  $(\underline{Q}^{1/2}, \underline{A})$  is always detectable.

$$\text{Now } \underline{K}\underline{A} + \underline{A}^T\underline{K} + \underline{Q} = \underline{0},$$

$$\text{so } \underline{K}[\underline{A} + (\underline{N} - \underline{M})\underline{K}] + [\underline{A} + (\underline{N} - \underline{M})\underline{K}]^T\underline{K} + 2\underline{K}\underline{M}\underline{K} + \underline{Q} - (\underline{K}\underline{N}\underline{K} + \underline{K}\underline{N}^T\underline{K}) = \underline{0}$$

$$\text{but } \underline{K}\underline{N}\underline{K} + \underline{K}\underline{N}^T\underline{K} = \underline{0} \text{ since } \underline{N} = -\underline{N}^T.$$

$$\text{If } \underline{K} \in LP^+(\underline{A}), \text{ then } \underline{Q} > \underline{0}$$

so  $(\sqrt{\underline{Q}} + 2\underline{K}\underline{M}\underline{K}, [\underline{A} + (\underline{N} - \underline{M})\underline{K}])$  is observable, which implies  $[\underline{A} + (\underline{N} - \underline{M})\underline{K}]$  is stable by Lemma 1 (ii). Otherwise, assume  $R(\underline{N}) \subset R(\underline{M})$ , which implies that there exists  $\underline{V}$  such that  $\underline{N} = \underline{V}\underline{M}$  or that  $(\underline{N} - \underline{M})\underline{K} = (\underline{V} - \underline{I})\underline{M}\underline{K}$ .

$$\text{By defining } \underline{B} \triangleq (\underline{V} - \underline{I})\underline{M}^{1/2}$$

$$\underline{F}^T \triangleq \underline{M}^{1/2}\underline{K}$$

$$\underline{P} \triangleq \underline{0}$$

$$\underline{R} \triangleq \underline{I}$$

in Lemma 1 (iii),  $[\sqrt{\underline{Q}} + 2\underline{K}\underline{M}\underline{K}, \underline{A} + (\underline{N} - \underline{M})\underline{K}]$  is detectable. By Lemma 1 (ii), we therefore have  $[\underline{A} + (\underline{N} - \underline{M})\underline{K}]$  stable.

QED

Remark:

A special case of Lemma 3 was established by Barnett and Storey (ref. 5). By specializing Lemma 3, we immediately obtain an explicit parametrization of a subclass of stabilizing feedback. First we introduce the following.

### Definition 3

Given the stable system  $\Sigma(\underline{A}, \underline{B})$ , let

$$S_1(\Sigma) \triangleq \{ \underline{G}^T \in \mathbb{R}^{m \times n} \mid \underline{G}^T = (\underline{S} - \underline{L})\underline{B}^T \underline{K}, \underline{S} = -\underline{S}^T, \underline{L} \geq \underline{0},$$

and either  $\underline{K} \in LP^+(\underline{A})$  or else

$$\underline{K} \in LP(\underline{A}) \text{ with } R(\underline{S}) \subset R(\underline{L}) \} \quad .$$

We can now state our result as

#### Theorem 1 -

Given the stable system  $\Sigma(\underline{A}, \underline{B})$ , then

(i)  $\underline{G}^T \in S_1(\Sigma)$  implies  $(\underline{A} + \underline{B}\underline{G}^T)$  is stable.

$$(ii) \quad \int_0^\infty e^{\underline{A}^T t} \underline{Q} e^{\underline{A} t} dt \geq \int_0^\infty e^{(\underline{A} + \underline{B}\underline{G}^T)^T t} \underline{Q} e^{(\underline{A} + \underline{B}\underline{G}^T) t} dt$$

where  $\underline{Q} \geq \underline{0}$  is such that  $\underline{K}\underline{A} + \underline{A}^T \underline{K} + \underline{Q} = \underline{0}$  and  $\underline{G}^T \triangleq (\underline{S} - \underline{L})\underline{B}^T \underline{K} \in S_1(\Sigma)$ .

Proof:

(i) Let  $\underline{M} = \underline{B}\underline{L}\underline{B}^T$ ,  $\underline{N} = \underline{B}\underline{S}\underline{B}^T$  in Lemma 3, and the result follows directly.

(ii) Let  $\underline{Q} \geq \underline{0}$  be such that

$$\underline{K}\underline{A} + \underline{A}^T \underline{K} + \underline{Q} = \underline{0}. (*)$$

Then we have

$$\underline{K} = \int_0^\infty e^{\underline{A}^T t} \underline{Q} e^{\underline{A} t} dt$$

Next rewrite (\*) as

$$\underline{K}(\underline{A} + \underline{B}\underline{G}^T) + (\underline{A} + \underline{B}\underline{G}^T)^T \underline{K} + (2\underline{K}\underline{B}\underline{L}\underline{B}^T \underline{K} + \underline{Q}) = \underline{0}$$

where  $\underline{G}^T \triangleq (\underline{S} - \underline{L})\underline{B}^T \underline{K} \in S_1(\Sigma)$  which implies

$$\underline{K} = \int_0^\infty e^{(\underline{A} + \underline{B}\underline{G}^T)^T t} (2\underline{K}\underline{B}\underline{L}\underline{B}^T \underline{K} + \underline{Q}) e^{(\underline{A} + \underline{B}\underline{G}^T) t} dt$$

hence

$$\int_0^{\infty} e^{\underline{A}^T t} \underline{Q} e^{\underline{A} t} dt = \int_0^{\infty} e^{(\underline{A} + \underline{B} \underline{G}^T)^T t} \underline{Q} e^{(\underline{A} + \underline{B} \underline{G}^T) t} dt \\ + 2 \int_0^{\infty} e^{(\underline{A} + \underline{B} \underline{G}^T)^T t} \underline{K} \underline{B} \underline{L} \underline{B}^T \underline{K} e^{(\underline{A} + \underline{B} \underline{G}^T) t} dt$$

or

$$\int_0^{\infty} e^{\underline{A}^T t} \underline{Q} e^{\underline{A} t} dt \geq \int_0^{\infty} e^{(\underline{A} + \underline{B} \underline{G}^T)^T t} \underline{Q} e^{(\underline{A} + \underline{B} \underline{G}^T) t} dt$$

QED

Remark: It can be easily shown that all eigenvalues of the feedback term  $\underline{B}(\underline{S} - \underline{L})\underline{B}^T \underline{K}$  have nonpositive real parts (term  $-\underline{B} \underline{L} \underline{B}^T \underline{K}$  has only real eigenvalues, while  $\underline{B} \underline{S} \underline{B}^T \underline{K}$  has only pure imaginary (conjugate pairs) eigenvalues or zero eigenvalues). This observation and the content of theorem 1(ii) makes it convenient to interpret  $S_1(\Sigma)$  as a natural generalization of the concept of "negative" feedback to the multivariable and multiinput case.

The next two corollaries are easy consequences of theorem 1.

Corollary 1.1 -

Let  $\Sigma(\underline{A}, \underline{B})$  be a system with a single input, that is, let  $\underline{B}$  be a column  $(n \times 1)$  vector  $\underline{b}$ . If  $\underline{g}_1^T, \dots, \underline{g}_j^T \in S_1(\Sigma(\underline{A}, \underline{b}))$ , then

$$\sum_{i=1}^j \alpha_i \underline{g}_i^T \in S_1(\Sigma) \quad \text{for all } \alpha_i \geq 0, \quad i = 1, \dots, j$$

Proof:

Each  $\underline{g}_i^T$  is of the form  $\underline{r}_i \underline{b}^T \underline{K}_i$  for some admissible  $\underline{r}_i, \underline{K}_i$ , so

$$\sum_{i=1}^j \alpha_i \underline{g}_i^T = \sum_{i=1}^j \alpha_i \underline{r}_i \underline{b}^T \underline{K}_i = \underline{b}^T \left( \sum_{i=1}^j \alpha_i \underline{r}_i \underline{K}_i \right)$$

But, from Lemma 2(i),  $\underline{K}_i \in LP(\underline{A})$  implies  $\sum_{i=1}^j \alpha_i \underline{r}_i \underline{K}_i \in LP(\underline{A})$  for all

$\alpha_i \underline{r}_i \geq 0$ ; hence  $\sum_{i=1}^j \alpha_i \underline{g}_i^T \in LP(\underline{A})$  for all  $\alpha_i \geq 0$ .

QED



Corollary 1.2 -

Suppose there exists  $\underline{L} \geq 0$  such that  $\underline{B}\underline{L}\underline{B}^T \in LP(\underline{A}^T)$ . Then  $[\underline{A} - \underline{B}\underline{L}\underline{B}^T(\underline{K} + \underline{N})]$  is stable for all  $\underline{K} \geq 0$  and  $\underline{N} = -\underline{N}^T$  such that  $R(\underline{K}) \supset R(\underline{N})$ .

If  $\underline{B}\underline{L}\underline{B}^T \in LP^+(\underline{A}^T)$  actually, then the condition  $R(\underline{K}) \supset R(\underline{N})$  can be omitted.

Proof:

Immediate from "taking the transpose" in Lemma 3.

QED

Theorem 1 has illustrated the importance of  $LP(\underline{A})$ . It is therefore useful to have an alternative characterization of  $LP(\underline{A})$ :

Proposition 1 -

$LP(\underline{A})$  is  $\underline{A}^T$ -invariant, that is, for all  $\underline{K} \in LP(\underline{A})$

$$\underline{A}^T R(\underline{K}) \subset R(\underline{K})$$

Proof:

$\underline{K} \in LP(\underline{A})$  iff  $\underline{K}\underline{A} + \underline{A}^T \underline{K} + \underline{H}\underline{H}^T = 0$  for some  $\underline{H}$ .

We claim that  $N(\underline{K}) = N(\underline{H}^T, \underline{A}) =$  unobservable subspace of  $(\underline{H}^T, \underline{A})$ .

For  $\underline{K} = \int_0^\infty e^{\underline{A}^T t} \underline{H}\underline{H}^T e^{\underline{A} t} dt$  so  $\underline{x} \in N(\underline{H}^T, \underline{A})$  implies  $\underline{H}^T e^{\underline{A} t} \underline{x} = 0$

for all  $t \in R$ , which implies  $\underline{x} \in N(\underline{K})$ . Conversely,  $\underline{x} \in N(\underline{K})$  implies

$$\int_0^\infty |\underline{H}^T e^{\underline{A} t} \underline{x}|^2 dt = 0 \text{ or } \underline{H}^T e^{\underline{A} t} \underline{x} = 0 \text{ for all } t \in R, \text{ that is,}$$

$$\underline{x} \in N(\underline{H}^T, \underline{A}).$$

To complete the proof, note that

$$\begin{aligned} R(\underline{K}) &= R(\underline{K}^T) = N(\underline{K})^\perp \\ &= N(\underline{H}^T, \underline{A})^\perp \\ &= R(\underline{A}^T, \underline{H}) \\ &= \text{controllable subspace of } (\underline{A}^T, \underline{H}) \end{aligned}$$

But any controllable subspace of  $\underline{A}^T$  is necessarily an  $\underline{A}^T$ -invariant subspace.

QED

Remark: The significance of proposition 1 is that it provides a systematic means for generating all members of  $LP(\underline{A})$ . For example, if  $\underline{A}$  has distinct real eigenvalues, then every  $\underline{K} \in LP(\underline{A})$  is of the form

$$\underline{K} = \underline{P}^T \underline{M} \underline{P}$$

where the rows of  $\underline{P}$  are left eigenvectors of  $\underline{A}$ , that is,

$$\underline{P} \underline{A} = \underline{\Lambda} \underline{P}, \underline{\Lambda} = \text{diagonal } (\lambda_1, \dots, \lambda_n)$$

and  $\underline{M} = \text{diagonal } (m_1, \dots, m_n)$ ,  $m_i \geq 0$ ,  $i = 1, \dots, n$ . Thus, all members of  $LP(\underline{A})$  can be trivially generated once  $\underline{P}$  is known.

While membership in  $S_1(\Sigma)$  is sufficient to guarantee closed-loop stability, it is, of course, not necessary, that is,  $S_1(\Sigma)$  is a strictly proper subset of  $S(\Sigma)$ . Intuitively, if the open-loop system is stable enough to begin with, it can tolerate a certain amount of "positive" feedback without leading to closed-loop instability. In other words, the poles of the open-loop system can be shifted to the right by feedback without destroying stability so long as none of them are shifted into the closed right-half plane. By allowing such additional nondestabilizing feedback, therefore, we ought to be able to "enlarge"  $S_1(\Sigma)$ . More precisely, we have the following.

#### Definition 4

Given the stable system  $\Sigma(\underline{A}, \underline{B})$  and any  $\underline{L} \geq \underline{0}$ ,  $\underline{L} \in R^{m \times m}$ , let

$$LP(\Sigma, \underline{L}) \triangleq \{\underline{K} \geq \underline{0} \mid \underline{K} \underline{A} + \underline{A}^T \underline{K} + 2 \underline{K} \underline{B} \underline{L} \underline{B}^T \underline{K} \leq \underline{0}\}$$

$$LP^+(\Sigma, \underline{L}) \triangleq \{\underline{K} \geq \underline{0} \mid \underline{K} \underline{A} + \underline{A}^T \underline{K} + 2 \underline{K} \underline{B} \underline{L} \underline{B}^T \underline{K} < \underline{0}\}$$

#### Definition 5

Given the stable system  $\Sigma(\underline{A}, \underline{B})$ , let

$$S_2(\Sigma) \triangleq \{\underline{G}^T \in R^{m \times n} \mid \underline{G}^T = (\underline{\hat{L}} + \underline{S}) \underline{B}^T \underline{K}, \underline{\hat{L}} = \underline{\hat{L}}^T, \underline{S} = -\underline{S}^T,$$

$$\underline{L} \geq \underline{0}, \underline{L} \geq \underline{\hat{L}}, \text{ and either } \underline{K} \in LP^+(\Sigma, \underline{L}) \text{ or else } \underline{K} \in LP(\Sigma, \underline{L}) \text{ with } R(\underline{\hat{L}} + \underline{S}) \subset R(\underline{L} - \underline{\hat{L}})\}.$$

#### Theorem 2 -

Given the stable system  $\Sigma(\underline{A}, \underline{B})$ , then  $\underline{G}^T \in S_2(\Sigma)$  implies  $(\underline{A} + \underline{B} \underline{G}^T)$  is stable.

Proof:

The proof follows by a straightforward extension of the proof of Lemma 3 and theorem 1 and hence is omitted.

QED

Remark: It can be easily seen that theorem 1 is just a special theorem 2 (with  $\underline{L} \equiv \underline{0}$  and  $\underline{\hat{L}} \leq \underline{0}$ ,  $S_2(\Sigma)$  will be reduced to that, in the general case covered by theorem 2, no definiteness made of  $\underline{\hat{L}}$ , and thus various "mixtures" of positive and negative allowed.

The next proposition provides further clarification on our tion scheme. First define

$$F_1(\underline{B}) \triangleq \{\underline{G}^T \in R^{m \times n} \mid \underline{G}^T = \underline{D}\underline{B}^T \underline{K}, \underline{D} \in R^{m \times n} \text{ arbitrary, } \underline{K} \in R^{n \times n} \text{ and } \underline{K} \geq \underline{0}\}$$

$$F_2(\underline{B}) \triangleq \{\underline{G}^T \in R^{m \times n} \mid \text{RK}(\underline{G}^T \underline{B}) < \text{RK}(\underline{G}^T)\}.$$

Proposition 2

$$F_1(\underline{B}) \cap F_2(\underline{B}) = \emptyset$$

$$F_1(\underline{B}) \cup F_2(\underline{B}) = R^{m \times n}$$

that is, any feedback map  $\underline{G}^T \in R^{m \times n}$  is either in the set  $F_1(\underline{B})$  or  $F_2(\underline{B})$ .

Proof:

We need only to show that

$$F_1(\underline{B}) = \{\underline{G}^T \in R^{m \times n} \mid \text{RK}(\underline{G}^T \underline{B}) = \text{RK}(\underline{G}^T)\}$$

Necessity: Suppose  $\underline{G}^T \in F_1(\underline{B})$ , that is, there exists  $\underline{D} \in R^{m \times n}$   $\underline{K} \in R^{n \times n}$ ,  $\underline{K} \geq \underline{0}$  such that  $\underline{G}^T = \underline{D}\underline{B}^T \underline{K}$ . Then

$$\underline{G}^T \underline{B} \underline{D}^T = \underline{D} \underline{B}^T \underline{K} \underline{B} \underline{D}^T \geq \underline{0}$$

$$\text{so } \text{RK}(\underline{G}^T \underline{B}) \geq \text{RK}(\underline{G}^T \underline{B} \underline{D}^T) = \text{RK}(\underline{D} \underline{B}^T \underline{K} \underline{B} \underline{D}^T) = \text{RK}(\underline{D} \underline{B}^T \underline{K}) = \text{RK}(\underline{G}^T).$$

Sufficiency:

Take  $\underline{D} = \underline{G}^T \underline{B}$  and observe that  $\underline{G}^T = \underline{G}^T \underline{B} \underline{B}^T \underline{K}$  has a solution  $\underline{K} = (\underline{B}^T \underline{B})^{-1}$  if  $\text{RK}(\underline{G}^T \underline{B}) = \text{RK}(\underline{G}^T)$ .

We now relate the content of proposition 2 to theorem 2. that  $S_2(\Sigma) \subset F_1$  and hence our parametrization scheme fails to nondestabilizing feedback map  $\in F_2(\underline{B})$ . That  $S(\Sigma) \cap F_2(\underline{B}) \neq \emptyset$  is illustrated by the following trivial example:

Example:

$$\underline{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad \lambda_1, \lambda_2 < 0, \quad \underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \underline{g}^T = [0 \ 1] \in F_2(\underline{b})$$

and  $(\underline{A} + \underline{b}\underline{g}^T) = \begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_2 \end{bmatrix}$  is stable.

Note, however, that if  $\underline{B}$  is of full rank, then the set  $F_2(\underline{B})$  is *not* generic in  $R^{m \times n}$ . The more interesting question — Is  $S_2(\Sigma)$  generic (i.e., dense) in  $S(\Sigma) \cap F_1(\underline{B})$ ? — is presently unsolved.

Our results so far have been on systems  $\Sigma(\underline{A}, \underline{B})$ , which are open-loop stable; the question next arises as to what the situation would be for systems *not* open-loop stable (i.e.,  $\underline{A}$  has unstable poles). For  $\underline{A}$  unstable, it is, of course, not possible to write Lyapunov-type equations. One is reminded, however, of the algebraic Riccati equations; indeed, we have the following interpretation of the traditional LQ-optimization problem:

#### Definition 6

Given  $(\underline{A}, \underline{B})$ , a stabilizable pair, let

$$LQ(\underline{A}, \underline{B}) \triangleq \{ \underline{K} \geq 0 \mid \underline{K} = \underline{K}(\underline{A}, \underline{B}, \underline{R}, \underline{H}^T) \text{ for some } \underline{R} > 0 \text{ and some } \underline{H}^T \text{ so that } (\underline{H}^T, \underline{A}) \text{ is a detectable pair} \}$$

where  $\underline{K}(\underline{A}, \underline{B}, \underline{R}, \underline{H}^T)$  denotes the unique positive semidefinite solution to the algebraic Riccati equation:

$$\underline{K}\underline{A} + \underline{A}^T \underline{K} - \underline{K}\underline{B}\underline{R}^{-1}\underline{B}^T \underline{K} + \underline{H}\underline{H}^T = 0$$

For  $\underline{R}$  fixed, we denote the corresponding set as  $LQ(\underline{A}, \underline{B}; \underline{R})$ .

#### Definition 7

$$S_3(\Sigma) \triangleq \{ \underline{G}^T \in R^{m \times n} \mid \underline{G}^T = -\underline{R}^{-1}\underline{B}^T \underline{K}, \underline{R} > 0, \underline{K} \in LQ(\underline{A}, \underline{B}; \underline{R}) \}$$

Proposition 3 -

Given any stabilizable system  $\Sigma(\underline{A}, \underline{B})$ ,  $\underline{G}^T \in S_3(\Sigma)$  implies  $(\underline{A} + \underline{B}\underline{G}^T)$  is stable.

Remark: The above proposition merely summarizes the well-known "standard" results of LQ-optimal feedback theory (refs. 1 and 4). However, the interpretation here of the LQ-optimal feedback class ( $S_3(\Sigma)$ ) as a parametrization of a subclass of stabilizing feedback is interesting.

Remark: It can be easily seen that theorem 1 is just a special case of theorem 2 (with  $\underline{L} \equiv \underline{0}$  and  $\underline{\hat{L}} \leq \underline{0}$ ,  $S_2(\Sigma)$  will be reduced to  $S_1(\Sigma)$ ). Note that, in the general case covered by theorem 2, no definiteness assumption is made of  $\underline{\hat{L}}$ , and thus various "mixtures" of positive and negative feedbacks are allowed.

The next proposition provides further clarification on our parametrization scheme. First define

$$F_1(\underline{B}) \triangleq \{\underline{G}^T \in R^{m \times n} \mid \underline{G}^T = \underline{D}\underline{B}^T\underline{K}, \underline{D} \in R^{m \times n} \text{ arbitrary,} \\ \underline{K} \in R^{n \times n} \text{ and } \underline{K} \geq \underline{0}\}$$

$$F_2(\underline{B}) \triangleq \{\underline{G}^T \in R^{m \times n} \mid \text{RK}(\underline{G}^T\underline{B}) < \text{RK}(\underline{G}^T)\}.$$

Proposition 2

$$F_1(\underline{B}) \cap F_2(\underline{B}) = \emptyset$$

$$F_1(\underline{B}) \cup F_2(\underline{B}) = R^{m \times n}$$

that is, any feedback map  $\underline{G}^T \in R^{m \times n}$  is either in the set  $F_1(\underline{B})$  or else  $F_2(\underline{B})$ .

Proof:

We need only to show that

$$F_1(\underline{B}) = \{\underline{G}^T \in R^{m \times n} \mid \text{RK}(\underline{G}^T\underline{B}) = \text{RK}(\underline{G}^T)\}$$

Necessity: Suppose  $\underline{G}^T \in F_1(\underline{B})$ , that is, there exists  $\underline{D} \in R^{m \times n}$  and  $\underline{K} \in R^{n \times n}$ ,  $\underline{K} \geq \underline{0}$  such that  $\underline{G}^T = \underline{D}\underline{B}^T\underline{K}$ . Then

$$\underline{G}^T\underline{B}\underline{D}^T = \underline{D}\underline{B}^T\underline{K}\underline{B}\underline{D}^T \geq \underline{0}$$

$$\text{so } \text{RK}(\underline{G}^T\underline{B}) \geq \text{RK}(\underline{G}^T\underline{B}\underline{D}^T) = \text{RK}(\underline{D}\underline{B}^T\underline{K}\underline{B}\underline{D}^T) = \text{RK}(\underline{D}\underline{B}^T\underline{K}) = \text{RK}(\underline{G}^T).$$

Sufficiency:

Take  $\underline{D} = \underline{G}^T\underline{B}$  and observe that  $\underline{G}^T = \underline{G}^T\underline{B}\underline{B}^T\underline{K}$  has a solution  $\underline{K} \geq \underline{0}$  if  $\text{RK}(\underline{G}^T\underline{B}) = \text{RK}(\underline{G}^T)$ .

QED

We now relate the content of proposition 2 to theorem 2. Observe first that  $S_2(\Sigma) \subset F_1$  and hence our parametrization scheme fails to capture any nondestabilizing feedback map  $\in F_2(\underline{B})$ . That  $S(\Sigma) \cap F_2(\underline{B}) \neq \emptyset$  is demonstrated by the following trivial example:

Example:

$$\underline{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad \lambda_1, \lambda_2 < 0, \quad \underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \underline{g}^T = [0 \quad 1] \in F_2(\underline{b})$$

and  $(\underline{A} + \underline{b}\underline{g}^T) = \begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_2 \end{bmatrix}$  is stable.

Note, however, that if  $\underline{B}$  is of full rank, then the set  $F_2(\underline{B})$  is not generic in  $R^{m \times n}$ . The more interesting question — Is  $S_2(\Sigma)$  generic (i.e., dense) in  $S(\Sigma) \cap F_1(\underline{B})$ ? — is presently unsolved.

Our results so far have been on systems  $\Sigma(\underline{A}, \underline{B})$ , which are open-loop stable; the question next arises as to what the situation would be for systems not open-loop stable (i.e.,  $\underline{A}$  has unstable poles). For  $\underline{A}$  unstable, it is, of course, not possible to write Lyapunov-type equations. One is reminded, however, of the algebraic Riccati equations; indeed, we have the following interpretation of the traditional LQ-optimization problem:

#### Definition 6

Given  $(\underline{A}, \underline{B})$ , a stabilizable pair, let

$$LQ(\underline{A}, \underline{B}) \triangleq \{ \underline{K} \geq 0 \mid \underline{K} = \underline{K}(\underline{A}, \underline{B}, \underline{R}, \underline{H}^T) \text{ for some } \underline{R} > 0 \text{ and some } \underline{H}^T \text{ so that } (\underline{H}^T, \underline{A}) \text{ is a detectable pair} \}$$

where  $\underline{K}(\underline{A}, \underline{B}, \underline{R}, \underline{H}^T)$  denotes the unique positive semidefinite solution to the algebraic Riccati equation:

$$\underline{K}\underline{A} + \underline{A}^T \underline{K} - \underline{K}\underline{B}\underline{R}^{-1}\underline{B}^T \underline{K} + \underline{H}\underline{H}^T = 0$$

For  $\underline{R}$  fixed, we denote the corresponding set as  $LQ(\underline{A}, \underline{B}; \underline{R})$ .

#### Definition 7

$$S_3(\Sigma) \triangleq \{ \underline{G}^T \in R^{m \times n} \mid \underline{G}^T = -\underline{R}^{-1}\underline{B}^T \underline{K}, \underline{R} > 0, \underline{K} \in LQ(\underline{A}, \underline{B}; \underline{R}) \}$$

Proposition 3 -

Given any stabilizable system  $\Sigma(\underline{A}, \underline{B})$ ,  $\underline{G}^T \in S_3(\Sigma)$  implies  $(\underline{A} + \underline{B}\underline{G}^T)$  is stable.

Remark: The above proposition merely summarizes the well-known "standard" results of LQ-optimal feedback theory (refs. 1 and 4). However, the interpretation here of the LQ-optimal feedback class  $(S_3(\Sigma))$  as a parametrization of a subclass of stabilizing feedback is interesting.

### III. STRUCTURAL STABILITY CHARACTERIZATION OF LINEAR QUADRATIC (LQ) OPTIMAL FEEDBACK MAPS

In this section, we show how the parametrization scheme developed previously can be applied to obtain a characterization of the closed-loop structural stability properties of systems under LQ-optimal feedback. More precisely, we establish an explicit parametrization of a general class of structural perturbations in the control feedback gains as well as in the control actuation matrix ( $\underline{B}$ ) that leave the closed-loop system stabilized. These new results, we believe, are the natural generalizations of some earlier results of Anderson and Moore (ref. 2).

We begin by first recalling from Lemma 2(iii) that, for  $\underline{A}$  stable,  $\underline{K} \in \text{LP}(\underline{A})$  always implies  $\underline{K} \in \text{LP}(\underline{A} - \underline{B}\underline{L}\underline{B}^T\underline{K})$ ; however, for  $\underline{A}$  unstable and  $\underline{K} \geq 0$  such that  $(\underline{A} - \underline{B}\underline{L}\underline{B}^T\underline{K})$  is stable, it need not be true that  $\underline{K} \in \text{LP}(\underline{A} - \underline{B}\underline{L}\underline{B}^T\underline{K})$ . The following example underscores this unfortunate state of affairs:

Example:

$$\underline{A} = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}, \quad \underline{B}\underline{L}\underline{B}^T = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad \underline{K} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Then  $(\underline{A} - \underline{B}\underline{L}\underline{B}^T\underline{K}) = \begin{bmatrix} -1 & 3 \\ 0 & -1 \end{bmatrix}$  is stable, but

$$\underline{K}(\underline{A} - \underline{B}\underline{L}\underline{B}^T\underline{K}) + (\underline{A} - \underline{B}\underline{L}\underline{B}^T\underline{K})^T\underline{K} = \begin{bmatrix} -2 & 3 \\ 3 & -2 \end{bmatrix} \not\leq 0$$

However, we have the following interesting observation.

Lemma 4:

$$\underline{K} \in \text{LQ}(\underline{A}, \underline{B}; \underline{R}) \Rightarrow \underline{K} \in \text{LP}(\underline{A} - \underline{B}\underline{R}^{-1}\underline{B}^T\underline{K})$$

Proof:

Immediate from the Riccati equation. QED

In other words, the above unfortunate state of affairs cannot occur if  $\underline{K}$  is an LQ solution.

We are now ready to state our first main result of the section.

Theorem 3 (infinite gain margin property):

Let  $\underline{K} \in \text{LQ}(\underline{A}, \underline{B}; \underline{R})$ . Then  $\{\underline{A} - [\underline{B}(\underline{S} + \underline{L})\underline{B}^T + \hat{\underline{B}}(\underline{N} + \underline{M})\hat{\underline{B}}^T]\underline{K}\}$  is stable for all  $\underline{L} \geq \underline{R}^{-1}$ ,  $\underline{M} \geq 0$ ,  $\underline{S} = -\underline{S}^T$ ,  $\text{R}(\underline{S}) \subset \text{R}(\underline{L} - \underline{R}^{-1})$

$$\underline{N} = -\underline{N}^T, \quad R(\underline{N}) \subset R(\underline{M})$$

$$\hat{\underline{B}} \text{ arbitrary}$$

Proof:

We have  $\underline{K} \in LP(\underline{A} - \underline{B}\underline{R}^{-1}\underline{B}^T\underline{K})$ , so by Lemma 3,  $[\underline{A} - \underline{B}\underline{R}^{-1}\underline{B}^T\underline{K} + (\underline{V} - \underline{W})\underline{K}]$  is stable for all  $\underline{W} \geq \underline{0}$ ,  $\underline{V} = -\underline{V}^T$  such that  $R(\underline{V}) \subset R(\underline{W})$ .

Take  $\underline{W} = \underline{B}(\underline{L} - \underline{R}^{-1})\underline{B}^T + \hat{\underline{B}}\underline{M}\hat{\underline{B}}^T$  and  $\underline{V} = \underline{B}\underline{S}\underline{B}^T + \hat{\underline{B}}\underline{N}\hat{\underline{B}}^T$  and we are done.

QED

Remark: For  $\hat{\underline{B}} \equiv \underline{0}$ , theorem 3 is a generalization of the "infinite gain margin" property of LQ-optimal feedback for single-input systems first noted by Anderson and Moore (ref. 1), who showed that the feedback gain vector  $\underline{g}^T = -(1/r)\underline{b}^T\underline{K}$  can be multiplied by any scalar  $\alpha \geq 1$  without destroying stability; the proof they used involves classical Nyquist techniques. Theorem 3 not only generalizes this property to multiinput systems, but allows more complicated alterations of the feedback gain vectors. Moreover, it makes the proof of this property much more transparent.

Remark: For  $\hat{\underline{B}} \neq \underline{0}$ , theorem 3 allows for changes in the  $\underline{B}$  matrix itself without destroying stability. One useful interpretation is the following.

Suppose that the optimal feedback gain matrix has been computed for a nominal  $\underline{B}_0$ , but that the actual value of  $\underline{B}$  during system operation is changed to  $\underline{B} = \underline{B}_0 + \underline{B}_1$ . Then the feedback term becomes  $(\underline{B}_0\underline{R}^{-1}\underline{B}_0^T\underline{K} + \underline{B}_1\underline{R}^{-1}\underline{B}_0^T\underline{K})$ . So long as  $\underline{B}_1 = \underline{B}_0(\underline{N} + \underline{M})\underline{R}$  for some  $\underline{N} = -\underline{N}^T$ ,  $\underline{M} \geq \underline{0}$ , theorem 3 guarantees that the system will remain stable (e.g.,  $\underline{B}_1 = \alpha\underline{B}_0$ ,  $\alpha > 0$ ). More complicated cases are allowed.

Remark: Alternatively, the case  $\hat{\underline{B}} \neq \underline{0}$  can be interpreted as allowing for the possibility of adding extra controllers, and using these extra feedbacks to "fine-tune" the closed-loop behavior of the original system. (A more systematic exploitation of this idea will be dealt with in a future publication; see also ref. 3).

Theorem 3 has dealt with the case when the "negative" feedback gains, etc., are allowed to increase in magnitude; the converse situation, when the "negative" feedback gains are reduced in magnitude (or when additional "positive" feedbacks are injected) is examined in the next proposition.

Theorem 4 (gain reduction and robustness property):

Let  $\underline{K} \geq \underline{0}$  be the Riccati solution to the LQ problem  $(\underline{A}, \underline{B}, \underline{R}, \underline{Q})$  where  $\underline{R} > \underline{0}$  and  $(\underline{Q}^{1/2}, \underline{A})$  detectable. Then

(i)  $[\underline{A} - \underline{B}(\underline{M} + \underline{N})\underline{B}^T\underline{K}]$  is stable for all  $\underline{M} > \underline{0}$  such that

$$\underline{M} > \frac{1}{2} \underline{R}^{-1} \quad \underline{N} = -\underline{N}^T$$



(ii) If  $(Q^{1/2}, A)$  is actually observable, then  $[A - B(M + N)B^T K + K^{-1}(\hat{Q} + \hat{N})]$  is stable where  $\underline{M}, \underline{N}$  are as above, and  $\hat{Q} = \hat{Q}^T$  is such that  $\hat{Q} < (1/2)Q$ ,  $R[(1/2)Q - \hat{Q}] \supset R(Q)$  and  $\hat{N} = -\hat{N}^T$  is such that  $R[(1/2)Q - \hat{Q}] \supset R(\hat{N})$ .

Proof:

(i) Let  $\Sigma_c \triangleq \Sigma[(A - BR^{-1}B^T K), B] = \Sigma(A_c, B)$ . Then we have  $K \in LP[\Sigma_c; (1/2)\underline{R}^{-1}]$  from the Riccati equation; so, by theorem 2,

$$[A_c + B(\hat{M} - N)B^T K] \text{ is stable for all } \hat{M} < \frac{1}{2} \underline{R}^{-1}$$

$$N = -N^T$$

or  $[A - B(\underline{R}^{-1} - \hat{M} + N)B^T K]$  is stable. Let  $\underline{M} \triangleq \underline{R}^{-1} - \hat{M} > (1/2)\underline{R}^{-1}$ , and the proof is complete.

(ii) Let  $\hat{A}_c \triangleq [A - B(M + N)B^T K]$ . From the Riccati equation, we have

$$K\hat{A}_c + \hat{A}_c^T K + KB(2M - R^{-1})B^T K + Q = 0.$$

Since  $(Q^{1/2}, A)$ -observable implies  $K > 0$ ,  $K^{-1}$  exists, so we have

$$K[\hat{A}_c + K^{-1}(\hat{Q} + \hat{N})] + [\hat{A}_c + K^{-1}(\hat{Q} + \hat{N})]^T K + KB(2M - R^{-1})B^T K + (Q - 2\hat{Q}) = 0$$

Hence, subject to the condition  $(1/2)Q \geq \hat{Q}$ ,  $R[(1/2)Q - \hat{Q}] \supset R(Q + \hat{N})$  it can be shown that

$$\left[ \sqrt{(Q - 2\hat{Q}) + KB(2M - R^{-1})B^T K}, \hat{A}_c + K^{-1}(\hat{Q} + \hat{N}) \right] \text{ is observable}$$

Thus, by Lemma 1(iii),  $[\hat{A}_c + K^{-1}(\hat{Q} + \hat{N})]$  is stable.

QED

Remark: Theorem 4(i) is a generalization of the known *gain reduction tolerance* property of LQ-optimal feedback. This interpretation is most transparent in the special case when  $\underline{R}^{-1} = \text{diag}(a_1, \dots, a_m)$  and  $\underline{M} = \text{diag}(\hat{a}_1, \dots, \hat{a}_m)$ ,  $\underline{N} \equiv 0$ . Then the original individual feedback loops are of the form  $\underline{u}_i = -a_i b_i^T K$ ,  $i = 1, \dots, m$ . The theorem states that, in this special case, the system remains stable if the feedback gains are reduced to  $\underline{u}_i = -\hat{a}_i b_i^T K$  so long as  $\hat{a}_i > (1/2)a_i$ . More complicated cases are, of course, allowed.

Remark: By interpreting the additional term  $K^{-1}(\hat{Q} + \hat{N})$  as a model perturbation term  $\delta A$  of the open-loop matrix  $A$ , we can use theorem 4(ii) to perform finite perturbation sensitivity analysis. The following simple example illustrates the usefulness of this approach.

Example:

$$\text{Let } \underline{A} = \begin{bmatrix} 0.5 & 0 \\ 0 & -2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

If we take

$$\underline{Q} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}, \quad R = \frac{1}{2}$$

Then we obtain the algebraic Riccati solution as

$$\underline{K} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and the optimal feedback gain  $\underline{g}^{*T} = -2[1 \ 1]$ . For any

$$\underline{\delta A} = \begin{bmatrix} \beta_1 & \beta_{12} + \gamma \\ \beta_{12} - \gamma & \beta_2 \end{bmatrix}$$

where

$$\gamma \in R, \quad \begin{bmatrix} \beta_1 & \beta_{12} \\ \beta_{12} & \beta_2 \end{bmatrix} < \begin{bmatrix} 0.5 & 1 \\ 1 & 3 \end{bmatrix}$$

we are assured, by theorem 4(ii), that

$$\underbrace{\begin{bmatrix} 0.5 + \beta_1 & \beta_{12} + \gamma \\ \beta_{12} - \gamma & -2 + \beta_2 \end{bmatrix}}_{\underline{A} + \underline{\delta A}} + \alpha \underline{b} \underline{g}^{*T}$$

is stable for all  $\alpha > 1/2$ . Consider the following special cases:

(a)  $\gamma = \beta_{12}$ ,  $\beta_1 = \beta_2 = 0$ . We have

$$\underbrace{\begin{bmatrix} 0.5 & 2\beta_{12} \\ 0 & -2 \end{bmatrix}}_{\underline{A} + \underline{\delta A}} + \alpha \underline{b} \underline{g}^{*T}$$

stable for all  $\alpha > 1/2$  and  $\beta_{12}$  so that

$$(1 - \beta_{12})^2 < 1.5 \text{ or } 1 - \sqrt{\frac{3}{2}} < \beta_{12} < 1 + \sqrt{\frac{3}{2}}$$

(b)  $\gamma = \beta_{12} = 0$ , we have

$$\begin{bmatrix} 0.5 + \beta_1 & 0 \\ 0 & -2 + \beta_2 \end{bmatrix} + \alpha \underline{b} \underline{g}^{*T}$$

stable for all  $\alpha > 1/2$  and  $\beta_1, \beta_2$  so that

- (i)  $\beta_1 < 0.5, \beta_2 < 3$
- (ii)  $(0.5 - \beta_1)(3 - \beta_2) > 1$

Thus if  $\beta_1 = 0$ , the perturbed system is stable for all  $\beta_2 < 1$ . Other more general cases are, of course, allowed.

The above example thus shows that the combined effect of feedback gain reduction and perturbation or uncertainty of the open-loop system parameters (poles and coupling terms) can be tolerated by a linear quadratic design without leading to closed-loop instability. This *robustness* property of the LQ-feedback design deserves more attention.

#### IV. CONCLUDING REMARKS

Since further applications of the parametrization results established here to reliable stabilization synthesis and decentralized stabilization coordination will be made in a future publication, we reserve a fuller discussion of the implications of our approach until then. However, we would like to point out an important implication for *practical* design that is immediate: the ability to perform feedback "loop-shaping" analysis.

In any realistic synthesis problem (keeping a system stabilized, localizing particular disturbances, etc.), there is usually a large number of feasible solutions. While the use of cost-criterion optimization (e.g., LQ) in theory allows the designer to pick exactly one such solution, in practice, the difficulties of judging or fully incorporating the relevant cost considerations and their tradeoffs as well as the often gross model uncertainties and physical variabilities of the system and the controllers necessitate further sensitivity analysis or trial-and-error "hedging" about the nominal solution. It is therefore very important in the computer-aided design context that the "feasible solution space" structure be known in some detail to facilitate and guide the conduct of iterative search. In this regard, a major merit of a "classical" design technique like root locus is that it provides an explicit functional dependence of the closed-loop system structures (distribution of poles and zeros) on the control structure (feedback gain). However, such classical approaches become totally intractable when there is a multiple number of controllers, while "modern," "state-space" linear feedback design techniques like "pole-placement" algorithms and "dyadic-feedback" design suffer the serious drawback of providing little structural information about the solutions they generate. Moreover, such techniques are guided more by mathematical convenience than by physical interpretation. From this perspective, the parametrization results established earlier show promise in providing the basis for a new iterative design algorithm that will overcome the last-mentioned drawbacks.

Several years ago, Rosenbrock (ref. 6) suggested a frequency-domain multiloop feedback design technique ("inverse Nyquist array" method) which he

motivated also as an attempt to overcome some of the abovementioned drawbacks. His approach is in contrast with ours, which is a "time-domain" approach. It will be interesting to investigate the connection, if any, between the two approaches.

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ANALYSIS OF PILOT-AIRCRAFT PERFORMANCE AND RELIABILITY  
VIA THE APPLICATION OF MAN-MACHINE MODELS

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SUMMARY

Analytic modeling of human performance has progressed to the point where it can be used with some confidence in the design and performance analysis of man-machine systems. This presentation first reviews the present approach to this analytic modeling, which utilizes human response theory together with modern control theory. Then an example of its use in helping design a flight director for the hover control of a VTOL vehicle is described. Finally, how the method could be used for analysis of human response to system failure is sketched.

## Objectives

- Develop and apply analysis procedures for manually controlled systems
- Understand human as an information processor, controller and decision maker

## Approach

- Combine human response theory with modern control and estimation theory

## Assumption

- Experienced human behaves optimally subject to his inherent limitations

## Applications to Manned Vehicle Systems

- Basic manual control tasks
- VTOL precision hovering
- Failure detection and control
- Carrier and land-based approach
- Anti-aircraft gunnery
- STOL flare and landing

Figure 1.- Modern control approach to human modeling.

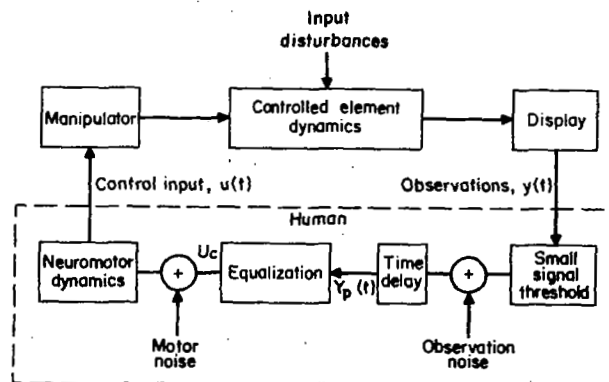


Figure 2.- Man as a generalized feedback control element.

$X(t)$  = Desired state trajectory

$x(t)$  = Deviation of system response from nominal

Linearization about  $X(t)$

$$\dot{x}(t) = Ax(t) + Bu(t) + \underbrace{Ew(t) + Fz(t)}_{\text{disturbances}}$$

$w(t)$  = Gaussian, zero mean white noise

$$E\{w(t)w(\tau)\} = W(t)\delta(t - \tau)$$

$z(t)$  = Non-random, bias inputs

$$\dot{h}(t) = A_h h(t) + F_h z(t)$$

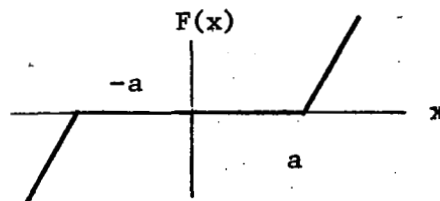
$Y(t)$  = Available information concerning  $x(t)$   
contains position and velocity information

$$Y(t) = Cx(t) + Du(t)$$

Figure 3.- Vehicle/display dynamics.

Lump sensory, central processing, perceptual limitations into perceptual equivalents

- Lumped human time-delay,  $\tau$
- Small signal thresholds,  $F(\cdot)$



- "Observation" and central processing noise,  $v_Y$

$Y_p(t)$  = "perceived" signal

$$Y_p(t) = F[Y(t - \tau)] + v_Y(t - \tau)$$

Figure 4.- Human perceptual limitations.

- Most important element in human randomness model
- Related to subject training level
- For foveal viewing of a single display

$$v_{yi}(t) = \pi \rho_{yi} E\{Y_i^2(t)\}$$

$\rho_{yi}$  = noise/signal ratio

- Scanning and attention allocation

$\eta_k$  = fractional attention to display  $k$

$$\eta_k \geq 0 ; \quad \sum_1^{NI} \eta_k = 1$$

$$\rho_{yi} = \rho_{yi}^0 / \eta_k$$

Human picks  $\eta_k$  to optimize information

Figure 5.- Equivalent observation noise.

- "Neuro-motor" dynamics;  $h(u) = u_c$

$$o(\ddot{u}) + T_N \dot{u} + u = u_c$$

$u_c$  = commanded control

$u$  = actual control

- Implicit modeling via limits on control rate
- "Motor" noise,  $v_u$

White, Gaussian noise

Second component of human randomness model

$$T_N \dot{u} + u = u_c + v_u$$

$$v_{ui}(t) = \rho_{ui} \cdot E\{u_i^2(t)\}$$

Figure 6.- Human control limitations.



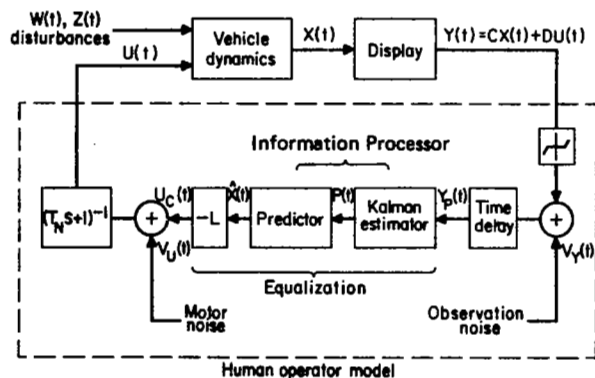


Figure 7.- Optimal control model of human response.

$$T_N \dot{u} + u = -L\hat{x}(t|t) + V_u(t)$$

$T_N$  = "neuromotor" lag from  $\dot{u}$  weighting

$L$  = optimal feedback gains

$\hat{x}(t|t)$  = best estimate of  $x(t)$  from  $\{Y_p(\sigma), \sigma \leq t\}$

Generation of  $\hat{x}(t|t)$  constitutes human's information processing behavior

- Kalman filter

Generates best estimate of  $\hat{x}(t - \tau|t)$

Compensates for  $v_Y(t)$

- Predictor

Generates  $\hat{x}(t|t)$  from  $\hat{x}(t - \tau|t)$

Compensates for delay

- Separable feedback control behavior

Figure 8.- Feedback control solution.

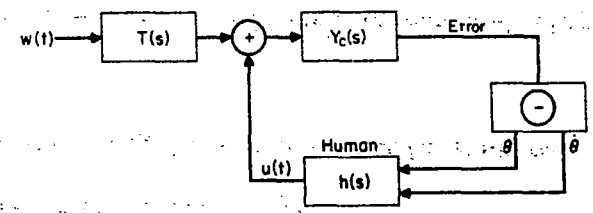
- Time delay,  $\tau$  0.15-0.25 sec
- Observation noise ratios,  $\rho_{yi} \cong 0.01$  (-20  $\pm$  3 dB)
- Thresholds,  $a_i$  0.05° visual arc (position)  
0.05°/sec (rate)
- Motor noise ratios,  $\rho_{ui} \cong 0.003$  (-25 dB)
- "Neuromuscular" lag,  $T_{Ni}$  0.08-0.12 sec

Assumption: Human parameters do not depend on vehicle dynamics or control cost functional

Human parameters do depend on external influences — light, heat, stress, vibration, display resolution, etc.

- Applications

Figure 9.- Typical human response parameter values.



- Typical  $Y_c = k, k/s, k/s^2$
- $T(s)$  = noise shaping filter
- $\dot{x}_1 = x_1 + w$        $y_1 = x_2$
- $\dot{x}_2 = x_1 + u$        $y_2 = \dot{x}_2 = x_1 + u$
- Human's task: minimize mean squared error

$$J(u) = E[y_1^2 + g \dot{u}^2]$$

- Steady-state, stationary behavior

$$u(s) = h_1(s)y_1(s) + h_2(s)y_2(s) + noise$$

$$= \underbrace{[h_1(s) + sh_2(s)]}_{\text{measurable}} y_1(s) + noise$$

Figure 10.- Simple SISO tracking tasks.

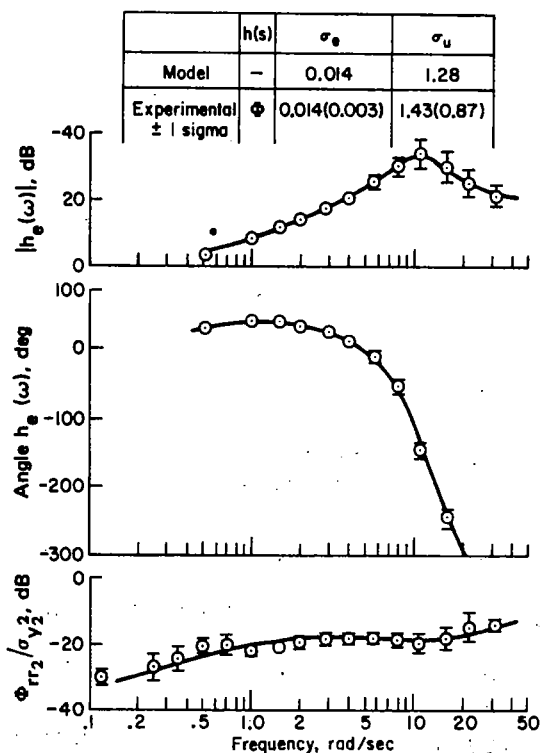


Figure 11.- Measured and predicted performable measures  $k/s^2$  dynamics (Avg of 3 subjects).

- Refinement of existing model

Analyze model-data mismatches

Effect of *a priori* information

Attentional allocation/scanning

- Human decision making

Decisions based on  $\hat{x}$

Fault detection

System monitoring

- Human adaptivity and learning

Explore relation to observation noise

Training effects

- Environmental effects

Relate to changes in human parameters

Stress, age, alcohol, vibration,  $g$ -forces

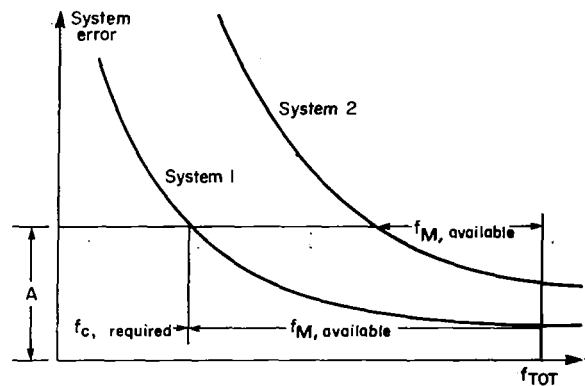
- Multi-human control tasks

- Human modeling in socio-economic systems

Figure 12.- Further efforts in man-modeling.

- Objective: allocate tasks between pilot and automatic system
- Approach:
  - 1) Select pilot/automatic configurations
  - 2) Determine information requirements for control and monitoring
  - 3) Perform comparative evaluation

Figure 13.- VTOL control/display design.



- Monitoring workload metric

$$f_M = \sum_i f_{M_i} = \text{monitoring workload}$$

$f_{M_i}$  = fractional monitoring attention  
on  $i$ th display

Figure 14.- Pilot monitoring with fixed total workload.

- Equal attention

$$f_{M_1} = f_M/n_y$$

Equivalent to residual monitoring

- Peak excursion

$$f_{M_1} \propto \text{prob } [|y_1| > k\sigma_1]$$

- Generalized quadratic index

$$J_M = \frac{1}{m} \sum_{i=1}^m \left( \frac{\gamma_i \sigma_{e_i}}{\sigma_{y_i}} \right)^2$$

- Instrument failure detection
- Relative estimation errors
- Performance assessment
- Miscellaneous

Figure 15.- Monitoring model concepts.

- Control performance predictions

$$J_c = \sum_i (\sigma_i/X_{i_{\max}})^2$$

- Control workload metric

$$f_c = \sum_i f_{c_i} = \text{control workload}$$

$f_{c_i}$  = fractional control attention on  $i$ th display element

- Attention allocation

$$\min_{f_{c_i}} J_c$$

Figure 16.- Optimal control pilot model.

$$\dot{x} = Ax + Bu_c + Ew, \quad y = Cx, \quad u_c = -L\hat{x}$$

$$J_c = J_c^0 + \text{tr}[L_e \Sigma L_e'] \quad L_e = \text{diag}(q_r^{1/2}) L_e^{Ar}$$

$$J_M = J_M^0 + \text{tr}[C_e \Sigma C_e'] \quad C_e = \text{diag}\left(\frac{\gamma_i}{y_i}\right) C_e^{Ar}$$

$$0 = \Sigma A' + A \Sigma + EWE' - \Sigma C' V^{-1} C \Sigma$$

Control:  $V_i = \rho_i \sigma_i^2 / f_{c_i} N_i$ ;  $\sigma_i = g(f_{c_1}, f_{c_2} \dots)$

Monitoring:  $V_i = \rho_i \sigma_i^2 / (f_{c_i} + f_{m_i}) N_i$ ;  $\sigma_i, f_{c_i} = \text{constant}$

Figure 17.- Expressions for cost measures  $J_c, J_M$ .

- Closed-form expressions for gradients

$$\frac{\partial J_c}{\partial f_{c_i}} \approx \frac{V_i}{f_{c_i}} \text{diag} \left[ G' \int_0^\infty e^{\hat{A}'\sigma} L_e' L_e e^{\hat{A}\sigma} d\sigma G \right]$$

$$\frac{\partial J_M}{\partial f_{m_i}} = \frac{V_i}{f_{m_i} + f_{c_i}} \text{diag} \left[ G' \int_0^\infty e^{\hat{A}'\sigma} C_e' C_e e^{\hat{A}\sigma} d\sigma G \right]$$

$$G = \Sigma C' V^{-1} = \text{Kalman filter gain}; \hat{A} = A - GC$$

- Nonlinear threshold effects included as RIDF
- Gradient projections

$$\Sigma f_i = \text{constant}; f_k = f_m \text{ for positions/rates}$$

- Basic gradient optimization for  $f_i$

Figure 18.- Algorithm for optimizing  $f_i$ .

- Longitudinal axis, pitch hold,  $f_c = 0.3$

- Predictions (A) without FD, (B) with FD

Instrument	$\sigma_{y_1}$		$f_{c1}/f_c$	
	w/o FD	FD	w/o FD	FD
Map	25.5 ft	16.6 ft	0.35	0.03
Altimeter	4.7 ft	4.7 ft	.17	.1
Pitch	1.0°	.7°	.03	.03
IVSI	1.4 fps	1.2 fps	.45	.13
FD <sub>x</sub>	---	.13 in.	---	.30
FD <sub>h</sub>	---	.14 in.	---	.40
$\delta_e$	.13 in.	.10 in.		
$\delta_c$	.25 in.	.22 in.		

Figure 19.- Helicopter hover task.

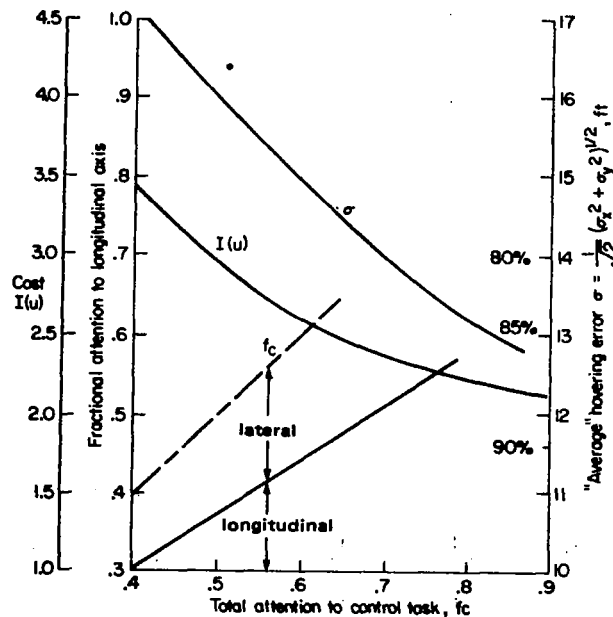


Figure 20.- Predicted performance CH-46 hover flight director used.

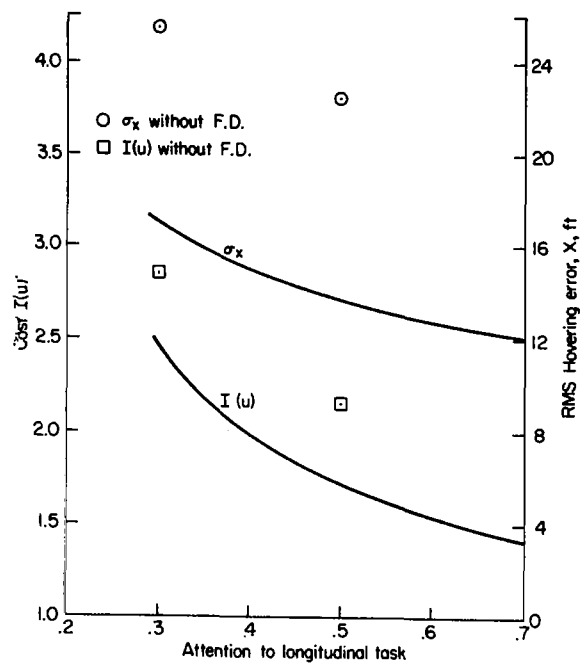


Figure 21.- Scores vs attention CH-46 hover longitudinal axis.

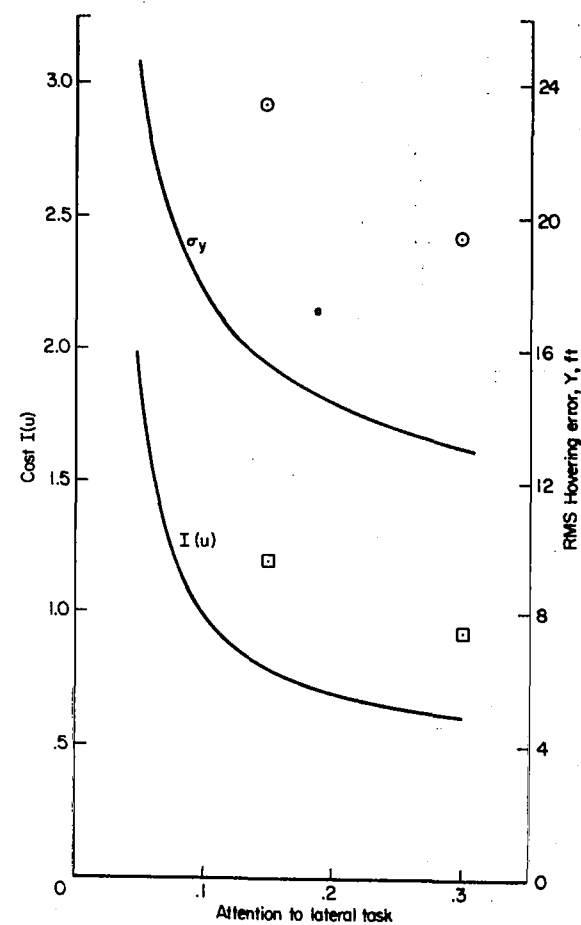


Figure 22.- Scores vs attention CH-46 hover lateral axis.



- Performance measures and workload metrics defined for control and/or monitoring
- Procedures developed for attention allocation - avoids previously arbitrary choice of  $f_1$
- Continuing efforts
  - Further validation
  - Determine information requirements
  - Tradeoff studies

Figure 23.- Summary.

#### Context

Detection and identification of unexpected system failures

#### Objective

Model the human's decision process and adaptive control behavior

#### Combine

- Human operator modeling
- Decision and control theory
- Human response results
- Planned experimentation

Figure 24.- Models for human decision-making and control.

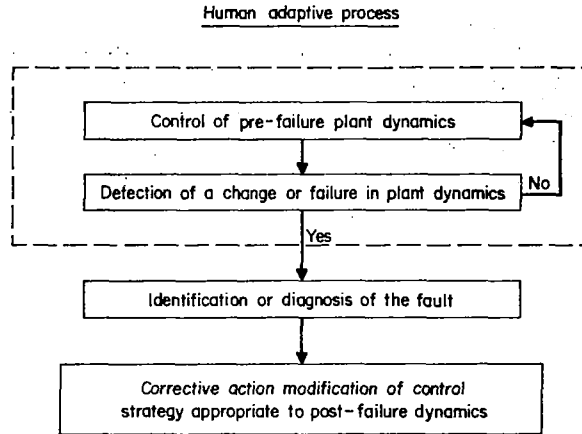


Figure 25.- Human adaptive process.

$$\left. \begin{array}{l} t < t_f : \quad \dot{x}(t) = A_0 x(t) + B_0 u(t) + w(t) \\ y(t) = C_0 x(t) \end{array} \right\} S_0$$

$$\left. \begin{array}{l} t > t_f : \quad \dot{x}(t) = A_k x(t) + B_k u(t) + w(t) \\ y(t) = C_k x(t) \end{array} \right\} S_k$$

$$S_k \in \{S_i, i = 0, 1, \dots, N\}$$

Humans task: Continuously determine true  $S_k$

- Models:
1. Sequential failure detection and identification
  2. Simultaneous failure detection and identification

Figure 26.- The adaptive control problem.

- Internal dynamic model in Kalman filter

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t - \tau) + G[y_p(t) - C\hat{x}(t)]$$

$$p(t) = \hat{x}(t - \tau)$$

- State estimate  $\hat{x}(t)$

$\hat{x}(t)$  = best internal estimate of vehicle state

$K_{(x-\hat{x})}(t)$  = error covariance

- Residual or innovations  $r(t)$

$$r(t) = y_p(t) - C\hat{x}(t - \tau)$$

Nominally  $r(t)$  = zero mean, white noise

Figure 27.- Features for adaptive modeling and decision making.

- If  $S_k = S_0$  then  $r(t|S_0)$  is zero-mean, white, with covariance  $v_y$

- Testing  $r(t)$  over  $t - T < \tau \leq t$

1. Mean value
2. Whiteness
3. Covariance

- Determine model parameters via experiment

- Limited model

Figure 28.- Failure detection by innovations testing.

- Model structure for control and decision
- Experimental planning, model validation
- Implications for automation
  1. Human monitoring
  2. Automatic status information
  3. Allocation of decision making

Figure 29.- Conclusions.

## RECENT PROGRESS IN MULTIVARIABLE CONTROL

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### SUMMARY

The last five years have witnessed a remarkable development in linear multivariable control, from both the theoretical and applied viewpoints. This paper summarizes the major new results in structural synthesis and design which are now available in textbook form, and suggests a number of promising directions for future research.

### 1. INTRODUCTION

The last five years have witnessed a remarkable development in linear multivariable control, from both the theoretical and applied viewpoints. Deferring for a moment the applied aspect, we note first that significant advances have taken place in our theoretical understanding of system structure. Under this heading we group two major topic areas: realization theory and system synthesis.

Realization theory is concerned with the axiomatic definition of "system," and with relating "internal" system descriptions (i.e., state descriptions) with "external" (i.e., input-output) descriptions. For this, a unified mathematical framework was supplied by Kalman *et al.* (ref. 1), exploiting the connections (well known to algebraists) between linear algebra and the theory of modules over a polynomial ring. The general theme has since been pursued in the context of delay systems (ref. 2), bilinear systems (ref. 3), and what might be called "categorical" systems (ref. 4).

While realization theory supplies the infrastructure, it is system synthesis and design that is perhaps of most direct relevance to engineers, and the remainder of this paper is devoted to sketching the situation here, together with the implications for practical applications. Before getting down to specifics, it may be useful to distinguish between "synthesis" and "design," at least in the context of control systems (cf. ref. 5, ch. 2).

I would label synthesis as the process by which one establishes the qualitative structural possibilities: that is, one determines whether and how one can achieve such desirable system properties as noninteraction (among functionally distinct subsystems and their associated controllers), loop stability, regulation of specified outputs with respect to disturbances, tracking by specified outputs to reference signals in some preassigned class,

and, finally, structural stability (i.e., preservation of the foregoing properties within a range of parameter variations).

On the other hand, design would refer to the numerical massaging (ideally, optimization) of system parameters, within the structural framework established by synthesis, to meet quantitative design specifications related (in this context) to transient response, stability margin, saturation levels, and so on. While the distinction is obviously not clear cut, we have at least two working definitions.

The point of making them is to stress that a control theory, *qua* theoretical construct, deals largely with synthesis, and thus may be expected to operate on a level of relative generality and abstraction. Its function is precisely to clarify the structural possibilities. Design may then bring in a host of special techniques, many of them heuristic, to ensure that a specific gadget works in practice. It is hoped that these simple-minded remarks may forestall useless arguments about the "gap."

We shall now review some recent theories of linear multivariable system synthesis, turning finally to their potential impact on design.

## 2. APPROACHES TO SYNTHESIS

A theory of synthesis will start from an assumed system model or class of models together with an assumed class of admissible controls, and supply constructive answers to a series of precisely posed questions of the following kind: Does an admissible control exist such that the resulting (synthesized) system displays such-and-such desirable characteristics? To qualify as a theory, and not just a library of algorithms, our constructive procedures should somehow exhibit the answers in terms of a reasonably small number of basic system concepts. While these must, of course, be made theoretically precise, they must also embody enough intuitive content to render the process of discovering new answers, by new procedures, easy and natural for the practitioner. Finally, the variety of questions amenable to attack must be wide enough to embrace a significant range of potential applications, and our constructive procedures must translate readily into computational algorithms: these will provide the framework for design, as noted earlier.

I think it is fair to claim that two complementary theories exist today which go some way to meeting these criteria. These are the geometric state-space approach (ref. 6 and references cited therein), and the polynomial matrix-frequency domain approach (ref. 5, 7, 8, and cited references).

I should emphasize that in neither of these theories is the notion of optimization at all central, and I omit discussion of the familiar "linear-quadratic" optimization problem, on which an extensive textbook literature already exists. It suffices to say that quadratic optimization is merely one method of achieving the rather limited goal of stabilization via state

feedback, and this theory, useful as it may be, contributes little to the primary objectives of structural synthesis.

## 2.1 Geometric State-Space Theory

Here we take for granted a (discrete or continuous-time) state model, of the familiar type

$$\dot{x} = Ax + Bu + Er + \dots$$

$$y = Cx + \tilde{C}r + \dots$$

$$z = Dx + \tilde{D}u + \dots$$

As usual,  $x$  and  $u$  denote the state and control vectors,  $r = r(t)$  is a disturbance vector,  $y$  is the vector of measured variables, and  $z$  the vector of variables to be regulated (e.g., tracking errors, deviations from set-point values). We restrict attention to the deterministic theory, although the stochastic counterpart is fairly straightforward.

In purely linear theory, the admissible controls are of the type

$$u = Fx + Gr + \dots$$

incorporating state feedback and (possibly) disturbance feedforward. There will be a constraint restricting the control to process only the measured variable  $y$ ; this translates algebraically as

$$\text{Ker}(F, G) \supset \text{Ker}(C, \tilde{C}) .$$

One can bring in observers or other dynamic compensators by various standard tricks.

What are the basic system concepts? Everything starts from the fundamental ideas of controllability and observability or, more accurately, the geometric objects "controllable subspace" and "unobservable subspace" of the state space  $x$ . Two other related families of subspace that play a basic role are the "(A,B)-invariant subspaces" and the "(A,B)-controllability subspaces."

To give a brief indication of their meaning, suppose we have simply

$$\dot{x} = Ax + Bu, \quad z = Dx .$$

Then we say that the subspace  $V \subset \text{Ker } D$  is (A,B)-invariant if, whenever the initial state  $x(0) \in V$ , there exists a control  $u(t)$  ( $t \geq 0$ ) such that the ensuing state trajectory  $x(t) = x(t; x(0), u)$  also satisfies  $x(t) \in V$  ( $t \geq 0$ ); then  $Dx(t) \equiv 0$ . It turns out that there is always a largest (A,B)-invariant subspace, say  $V^*$ , contained in  $\text{Ker } D$ : roughly, this is the largest set of states which can be made unobservable (by suitable

control) at the output  $z$ . Finally, the subspace  $R \subset X$  is an  $(A,B)$ -*controllability subspace* if every  $x \in R$  can be reached from the origin of  $x$  by a controlled state trajectory wholly contained in  $R$ : thus, if  $R \subset \text{Ker } D$ , you can sneak from zero to any  $x \in R$  without being seen from  $z$ . There is always a largest  $R$ , say  $R^*$ , in  $\text{Ker } D$ , and we always have  $R^* \subset V^*$ .

Using just these natural and elementary ideas, it has been possible to build up an elaborate set of procedures for structural synthesis. What problems have been solved?

Historically (1970), the first was the longstanding problem of noninteracting control or decoupling. In the simplest version, one is given the system

$$\dot{x} = Ax + Bu$$

with output vectors

$$z_i = D_i x, \quad i = 1, \dots, k$$

One asks for a state feedback control with new external inputs  $v_i$ ,

$$u = Fx + \sum_{i=1}^k G_i v_i,$$

so that each  $v_i$  controls the output  $z_i$  alone without affecting the  $z_j$  for  $j \neq i$ . Under very reasonable conditions on the number of available controls, one can show that this is almost always possible (at least by dynamic compensation, omitted here); furthermore, one can achieve more-or-less arbitrary pole locations for the closed-loop system. The condition for solvability is just that

$$R_i^* + \text{Ker } D_i = x, \quad i = 1, \dots, k$$

where  $R_i^*$  is the largest controllability subspace contained in

$$\bigcap_{j \neq i} \text{Ker } D_j$$

The numerical checking of this condition is quite simple. At the University of Toronto, we have developed APL routines that have been run successfully on decoupling problems of at least moderate dynamic order (15-20).

The second major structural problem to be treated completely and successfully by these methods was the general multivariable servoregulator problem (1974). One is given

$$\dot{x} = Ax + Bu, \quad y = Cx, \quad z = Dx$$

Here the dynamics represent the plant together with a dynamic model of the disturbance (and/or reference) signals that must be rejected (and/or tracked).



One asks for a control  $u = \tilde{F}y$  so that  $z$  is regulated ( $z(t) \rightarrow 0$  from any initial state) and so that the loop is stable. While these two requirements are not necessarily compatible, it turns out that they almost always are, provided one has at least as many scalar controls as scalar variables to be regulated. The precise solvability conditions are a little too involved to state here, but they amount to quite transparent relations among suitably defined  $V^*$ ,  $R^*$ . Again, it is straightforward to develop a synthesis procedure that translates, for example, to APL (ref. 9).

Limitations of space prohibit going into further detail: may I just mention that the geometric theory has also shed light on decentralized stabilization (ref. 10) and to various synthesis problems involving system "zeros" (ref. 11). In summary, the "geometric approach" has already justified itself in terms of results and shows ample promise for future development.

## 2.2 Polynomial Matrix Formalism

On taking Laplace transforms of the equations

$$\dot{x} = Ax + Bu$$

$$z = Dx + \tilde{D}u$$

one is led to the *polynomial system matrix*:

$$P(s) = \begin{bmatrix} A-sI & B \\ D & \tilde{D} \end{bmatrix}$$

More generally, polynomial system matrices of form

$$P(s) = \begin{bmatrix} T(s) & U(s) \\ -V(s) & W(s) \end{bmatrix}, \quad \det T(s) \neq 0$$

arise when higher order differential operators appear in the initial dynamic description. The more familiar *transfer matrix* is now defined as

$$G(s) = V(s)T^{-1}(s)U(s) + W(s)$$

The rich structure theory of  $P(s)$  and  $G(s)$  has been explored notably by Rosenbrock (refs. 5 and 7) and Wolovich (ref. 8). As these authors adopt somewhat different viewpoints, we shall review their approaches separately. Of course, in the space available we cannot do them justice.

In terms of  $P(s)$ , Rosenbrock supplies and justifies the appropriate definition of system dynamic order ( $= \deg [\det T(s)]$ ), together with that of system poles and zeros, and relates these concepts to the controllability-observability structure of a state variable realization. By the introduction of "strict system equivalence" (based on coordinate transformations involving

no change in dynamic order), it is shown how  $P(s)$  can be reduced to various useful standard forms; and then how the invariant structure of  $P(s)$  relates to that of  $G(s)$  (e.g., the invariant polynomials of  $P$  coincide with the Smith-McMillan numerator polynomials of  $G$ ). (In this review, some accuracy is sacrificed to brevity.) The system matrix is further exploited to discuss open-loop zero assignment (by output selection) and closed-loop pole assignment (by state feedback).

The formal theory culminates with an investigation of the key idea of diagonal dominance. A rational matrix  $Q(s) = [q_{ij}(s)]$  is *diagonally (row) dominant* on a contour  $C$  in the complex  $s$ -plane if

$$|q_{ii}(s)| - \sum_{j \neq i} |q_{ij}(s)| > 0$$

for all  $i$ , and  $s \in C$ . Rosenbrock now shows that if the forward-loop transfer matrix is diagonally dominant (with  $C$  a standard Nyquist path), then stability of the closed-loop system can be studied by application of the Nyquist (or inverse Nyquist) criterion to the diagonal elements alone. Coupling effects are accounted for by broadening the usual inverse Nyquist loci into "bands," obtained by ingenious exploitation of various inequalities due to Gershgorin and Ostrowski.

While something will be said below about the design methods that ensue, this summarizes the "synthetic" content of Rosenbrock's approach (refs. 5 and 7).

What are the basic system concepts? — again, controllability and observability; next, the polynomial system matrix and its equivalent forms; and, finally, diagonal dominance or quasi-noninteraction.

What synthesis problems have been solved? Actually Rosenbrock does not pose synthesis problems in a highly formal way, but proceeds more in the spirit of "design" to illustrate how the foregoing concepts can be exploited to satisfy various closed-loop requirements. Heuristics are proposed to achieve open-loop diagonal dominance through simple types of plant precompensation. Here interactive computation is essential, with graphical display of inverse Nyquist arrays. The result is to reduce the design problem to decoupled "classical" subproblems for independent single-input/single-output loops. It is reported (convincingly) that criteria of stability margin, bandwidth, structural integrity, and so forth can be effectively brought into play.

Turning now to Wolovich's approach (ref. 8), we find a creditable attempt to unify the state and frequency viewpoints. The starting point is a "structure theorem" that represents a transfer matrix  $G(s)$  in the form

$$G(s) = N(s)D^{-1}(s)$$

where  $N(s)$  and  $D(s)$  are polynomial matrices that satisfy a certain coprimeness relation. Under the action of linear state-variable feedback, it turns out that the "numerator" matrix  $N(s)$  remains unaltered, while the "denominator" matrix  $D(s)$  is almost freely assignable. In this way, the effect of a time-domain compensation scheme can be displayed in the frequency domain. A further result shows that any control scheme that can be implemented by feedforward compensation can also be implemented via an equivalent feedforward/feedback scheme.

With these as the basic concepts, Wolovich turns to a series of synthesis problems, formally posed in terms of transfer matrices. The major problems are noninteracting control (decoupling) and exact model matching. The decoupling problems are resolved by constructive procedures; for model matching, heuristics are proposed. It is worth remarking that, while Wolovich's expressed intent is to unify the state and frequency approaches, his transfer matrices are treated as purely algebraic objects: the usual frequency domain ideas (e.g., bandwidth, gain, and phase margins) never appear at all. In effect, his procedures represent disguised linear algebra, with the polynomials serving mainly as bookkeeping devices. Of course, this is not to say that the procedures, once mastered, may not be effective; presumably, they are most convenient when system data are supplied initially in transfer matrix form.

### 3. IMPACT ON DESIGN

My remarks here must be brief and tentative, as the methods just described are quite recent. One thing is clear: as the methods have been evolved for application to large, complex systems, design will rely on interactive computing, presumably with graphical display (of Nyquist arrays, pole-zero patterns, time and frequency responses, etc.). For this, development of efficient numerical algorithms and their organization via a high-level procedural language is the kind of major advance to be expected in the near-term future. Considerable progress has already been achieved by, for instance, Rosenbrock at the University of Manchester (U.K.), Åström at the Lund Institute of Technology (Sweden), and Mansour at the ETH (Zurich). These three groups are sustained in part by substantial interaction with industry. Mention should also be made of the work in computer-aided design at Cornell, as reported by Merriam (ref. 12).

### 4. CONCLUSION

Large-system multivariable synthesis has arrived. The outlook for attacking the further problems of hierarchical and decentralized control is good. The effective exploitation of what is known even now will, nevertheless, demand patience, and commitment of resources.

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## A SUMMARY OF NONLINEAR FILTERING

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### I. INTRODUCTION

In this summary paper, we attempt to highlight the main results of modern nonlinear filtering theory and discuss some of the approximations commonly made to yield suboptimal filter designs (such as extended Kalman filters) that avoid the infinite-dimensional on-line computations required of the optimal nonlinear filter. No attempt is made to provide a survey of the field of nonlinear filtering or even to begin to approach the breadth and depth of coverage or the exhaustiveness of referencing a survey implies. Nor does this paper provide the self-contained development conventionally thought of as being implicit in a tutorial introduction. We content ourselves with outlining the main approaches, stating and discussing the main results, and referring to a representative sampling of papers or books in which derivations, proofs, details, and extensions can be found. These considerations plus a desire to be brief leave great latitude in the selection and relative emphasis of topics and, perhaps even more than most papers of a summary or survey type, this paper reflects a personal view of the field of modern nonlinear filtering.

This paper is organized as follows: section II is devoted to the formulation of the prototype nonlinear filtering problem with which the rest of the paper is concerned, along with a discussion of some of the modifications, extensions, and alternative models found in the literature. In section III, we outline the main approaches used to characterize or find the optimum nonlinear filter; in section IV, we recall the enormous simplifications that result when the system is linear. Sections V and VI are concerned with approximations to the optimal nonlinear filter: the former with relinearized estimators and extended Kalman filters, the latter with approximations to the conditional density. Finally, section VII summarizes an approach to designing and evaluating the performance of suboptimal filters in terms of upper and lower bounds.

### II. FORMULATION OF NONLINEAR FILTERING PROBLEM

For the nonlinear filtering problems considered here, attention will be concentrated on the smooth, finite-dimensional, continuous-time dynamic system:

$$dx_t = f(x_t, t)dt + D(t)dv_t \quad (1a)$$

$$dz_t = g(x_t, t)dt + G(t)dw_t \quad (1b)$$

where  $x_t$  is an  $n$ -dimensional random vector,  $v_t$  is  $r$ -dimensional,  $z_t$  and  $w_t$  are  $m$ -dimensional, and  $v$  and  $w$  are independent normalized Wiener processes. The initial state  $x_0$  is independent of  $v$  and  $w$  and is Gaussian with mean  $\hat{x}_0$  and covariance  $\Sigma_0$ . Functions  $f$  and  $g$  are assumed to satisfy suitable technical properties;  $G(t)$  is nonsingular. Everything takes place on the fixed finite interval  $[0, T]$ . Less rigorously, we can also write

$$\dot{x}_t = f(x_t, t) + D(t)\dot{v}_t \quad (1a')$$

$$y_t = \dot{z}_t = g(x_t, t) + G(t)\dot{w}_t \quad (1b')$$

where the formal derivatives  $\dot{v}$  and  $\dot{w}$  of the Wiener processes  $v$  and  $w$  are (independent) Gaussian white noises. The system equation (1') can be obtained formally by dividing both sides of equation (1) by  $dt$  and thinking of  $y = \dot{z}$  as the observed processes. Similar formal manipulations can be used to convert, for example, the filters written later in the "differential" notation used in equation (1) to the "derivative" form used in equation (1').

We will also restrict attention to minimum-mean-square-error filtering. Letting  $Z_t$  be the past of the observation process  $z$ ,<sup>1</sup> that is,

$$Z_t = \{(z_s, s), \quad 0 \leq s \leq t\} \quad (2)$$

we are thus interested in finding the conditional mean

$$\hat{x}_t \triangleq E[x_t \mid Z_t] \quad (3)$$

of  $x_t$  given  $Z_t$ , and the corresponding conditional covariance

$$\Sigma_t = \text{cov}[x_t \mid Z_t] = E[(x_t - \hat{x}_t)(x_t - \hat{x}_t)' \mid Z_t] \quad (4)$$

Throughout the paper, prime denotes transposition. The conditional covariance provides, of course, a measure of accuracy or confidence.

The filtering problem defined by equations (1) to (4) has established motivation as a model that arises in a wide variety of control and communication problems, and most of the literature in the systems area is concerned with this model or a modification of it. Some of the many possible modifications and extensions include the following.

(a) From a control viewpoint, the most important extension is to replace equation (1a) by

$$dx_t = f(x_t, u_t, t)dt + D(t)dv_t \quad (1a'')$$

where the ( $r$ -dimensional) control  $u_t$  is assumed to be derived causally from the output process  $z$ . Most of the results given later have natural counterparts for this model.

---

<sup>1</sup>More precisely,  $Z_t$  is the  $\sigma$ -algebra generated on the underlying probability space by the observation process up to time  $t$ .

(b) Smoothing and prediction problems, where interest centers on finding  $E[x_t | Z_s]$ ; prediction problems have  $s < t$ , smoothing ones have  $s > t$ . More generally, one can seek  $E[h(x_t) | Z_s]$  for some suitable function  $h$ , and we can have  $s > t$ ,  $s < t$ , or  $s = t$ .

(c) The finite-dimensional continuous-time system (1) can be replaced by alternative system models; for example, by a finite-dimensional discrete-time system

$$x_{k+1} = f(x_k, k) + D(k)v_k \quad (5a)$$

$$z_k = g(x_k, k) + G(k)w_k \quad (5b)$$

or by an infinite-dimensional system such as a delay-differential equation or a partial differential equation (refs. 1-6). Consideration of the discrete-time finite-dimensional model (5) avoids a number of technical mathematical questions associated with the continuous-time system (1); conversely, technical problems are accentuated for infinite-dimensional models, even when they are specialized to being linear.

(d) Generalization of noise process models: extensions are available for correlated noise processes  $v$  and  $w$  ( $\dot{v}$  and  $\dot{w}$ ), "colored" (rather than "white")  $\dot{v}$  and  $\dot{w}$ , and state-dependent noises where the distribution matrices  $D(t)$  and  $G(t)$  in equation (1) depend also on the system state. When  $f$  and  $g$  are linear, a "wide-sense" approach can be added: here  $v$  and  $w$  are taken to be uncorrelated-increment processes ( $\dot{v}$  and  $\dot{w}$  wide-sense white) not necessarily Gaussian, and one seeks the *linear* minimum-mean-square-error estimator. Problems in which  $G(t)$  is singular lead to singular estimation problems: only when  $f$  and  $g$  are linear has some headway been made (refs. 1, 5, and 7-11).

(e) Different criteria: the minimum-mean-square-error optimality criterion can be replaced by other choices, such as maximum *a posteriori* (MAP) estimation, etc. Other modifications include formulating the problem as one of statistical least-squares estimation; invariant imbedding or the calculus of variations are typical solution techniques (e.g., refs. 12 and 13).

(f) Alternative observation models arise naturally in some applications. For example, for certain problems in nuclear medicine and quantum-limited optical communication, the observations take the form of a point (counting) process usually modelled as a Poisson process whose intensity (rate) is itself a random process influenced by the system state  $x_t$  (e.g., refs. 14 and 15). More recently, martingale theory has been used to provide a unified treatment that includes counting process models and the model (1) as special cases within a more general setting (e.g., refs. 16 and 17).

(g) Special structures: For some classes of systems, a vector space setting is less natural than a formulation in some other mathematical framework whose structure is better matched to the characteristics of the class of systems at hand. An example is the formulation of some problems on manifolds

and bringing to bear the techniques of differential geometry in attacking them. Included here are estimation problems involving bilinear systems (refs. 18-20).

(h) Combination problems: Estimation problems frequently arise in combination with additional requirements such as identification or hypothesis-testing (the latter includes detection and some formulations of reliability problems). Superimposed on top of these can also be a control task.

### III. APPROACHES TO SOLVING NONLINEAR FILTERING PROBLEM

There are essentially two main analytical approaches to finding or characterizing the conditional mean and conditional covariance. The first is to find or characterize the conditional density  $p_t(x_t | Z_t)$  of  $x_t$  given  $Z_t$ , or the conditional distribution  $\mu_t(x_t | Z_t)$ . The conditional mean and covariance then follow as

$$\hat{x}_t = \int_{R^n} x p_t(x | Z_t) dx = \int_{R^n} x d\mu_t(x | Z_t) \quad (6)$$

$$\begin{aligned} \Sigma_t &= \int_{R^n} (x - \hat{x}_t)(x - \hat{x}_t)' p_t(x | Z_t) dx \\ &= \int_{R^n} (x - \hat{x}_t)(x - \hat{x}_t)' d\mu_t(x | Z_t) \end{aligned} \quad (7)$$

There are, in turn, two main ways in which  $p_t(x_t | Z_t)$  can be characterized.

(A)  $p_t(x_t | Z_t)$  satisfies the partial differential equation

$$d_t p_t(x | Z_t) = L^*[p_t(x | Z_t)]dt + p_t(x | Z_t)[g(x, t) - \hat{g}_t]'W^{-1}(t)[dz_t - g_t dt] \quad (8)$$

where  $\Delta d_t$  on the left side denotes differential with respect to time,  $W(t) = G(t)G'(t)$ ,

$$\hat{g}_t = E[g(x, t) | Z_t] = \int_{R^n} g(x, t) p_t(x | Z_t) dx \quad (9)$$

and  $L^*$  is the forward Kolmogorov partial differential operator associated with the system (1):

$$L^*[\cdot] = - \sum_{i=1}^n \frac{\partial [f_i(x, t) \cdot]}{\partial x_i} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2}{\partial x_i \partial x_j} [D(t)D'(t) \cdot]_{ij} \quad (10)$$

The stochastic partial differential equation (8) has been called Kushner's equation (refs. 21 and 22). It is also referred to as a modified Fokker-Planck equation or a conditional Kolmogorov equation.



The practical difficulties in attempting to find  $p_t(x_t | Z_t)$  from equation (8) are the obvious computational problems of solving a partial-differential equation on line. Levieux (ref. 23) reported recently on his efforts in this direction; indications are that the approach is limited with current computational technology to a state  $x_t$  of dimension at most two or three.

(B) A second way to characterize  $p_t(x_t | Z_t)$  is via the so-called Bucy representation theorem (ref. 1) that relates the conditional density of  $x_t$  given  $Z_t$  to the prior density  $p_t(x_t)$  of  $x_t$ :

$$p_t(x | Z_t) = \frac{E\{\exp \phi_t | Z_t, x_t = x\}}{E\{\exp \phi_t | Z_t\}} p_t(x) \quad (11)$$

where (log likelihood ratio)  $\phi_t$  is

$$\phi_t = \int_0^t g'(x_s, s) dz_s - \frac{1}{2} \int_0^t \|g(x_s, s)\|_{W^{-1}(s)}^2 ds \quad (12)$$

The representation (11) can be viewed as a form of the Bayes rule suited to the problem under consideration here. Its validity rests on certain conditional independence assumptions satisfied for the prototype problem formulated in section II, but are violated if, for example,  $v$  and  $w$  are correlated or if equation (1a'') is substituted for (1a). Note that equation (11) is not as it stands an explicit formula from which  $p_t(x | Z_t)$  can be calculated but rather it is a relation that  $p_t(x | Z_t)$  must satisfy since the coefficient of  $p_t(x)$  on the right side depends implicitly on  $p_t(x | Z_t)$ . Note also that the numerator of this coefficient involves a conditional expectation in which attention must be confined to state trajectories that pass through point  $(x, t)$ . We return in section VI to the use of representation (11) and its variations in constructing approximations to  $p_t(x | Z_t)$ .

As an alternative to first finding the conditional density and using equations (6) and (7) to compute the conditional mean and covariance, one can instead write stochastic differential equations for  $x_t$  and  $\Sigma_t$ :

$$d\hat{x}_t = \hat{f}_t dt + [\hat{x}_t g'_t(x_t, t) - \hat{x}_t \hat{g}'_t] W^{-1}(t) [dz_t - \hat{g}_t dt] \quad (13)$$

$$d\Sigma_t = \overbrace{f(x_t, t)[x_t - \hat{x}_t]' dt} + \overbrace{[\hat{x}_t - \hat{x}_t] f'(x_t, t) dt} - d\hat{x} d\hat{x}' + D(t) D'(t) dt \\ + \overbrace{[\hat{x}_t - \hat{x}_t][x_t - \hat{x}_t]' [g(x_t, t) - \hat{g}_t] W^{-1}(t) [dz_t - \hat{g}_t dt]} \quad (14)$$

where  $\hat{f}_t = E[f(x_t, t) | Z_t]$ , etc. These stochastic differential equations can be derived in a number of ways, such as by combining equations (6) and (8) and integrating by parts, by integrating equation (11) by parts, by the innovations approach (ref. 7), or by measure transformation approaches (refs. 24-26).

However, the integration of equation (13) requires the simultaneous computation of  $\hat{f}_t$ ,  $\hat{g}_t$ , and  $\bar{x}_t g'_t(x_t, t)$  and, although stochastic differential equations for each of these can also be written down, they, in turn, require simultaneous calculation of even more terms such as  $\dot{g}_t g'_t = E[g(x_t, t)g'(x_t, t) | Z_t]$ , the stochastic differential equations for which require even more terms, and so ad infinitum. Similar remarks apply to  $\Sigma_t$ . Thus, except in certain special cases where this mushrooming requirement for additional terms terminates at a finite number, we are left with the need to solve an infinite system of stochastic differential equations.

Thus, generally all three approaches to computing the conditional mean and covariance require an infinite-dimensional on-line calculation, whether that is the determination of the conditional density from the partial differential equation (8) or via representation (10), or the solution of the infinite system of stochastic differential equations in which equations (13) and (14) are imbedded. Each provides its own perspective on the common conclusion: the exact optimum filter is infinite-dimensional and therefore computationally infeasible. Our interest then turns naturally to suboptimum but practically implementable filters and, with it, what each of the three approaches offers in suggesting suboptimum estimator structures and design procedures and in providing a basis for error analysis (performance evaluation). Before turning to this, however, we review briefly the special case of a linear system, for which the optimum filter happens to be finite-dimensional. Apart from its intrinsic interest as a filtering problem with an exact, implementable solution, the linear case serves both as a benchmark and as a source of potential structures for suboptimal candidate designs.

#### IV. LINEAR FILTERING PROBLEMS

When the system (1) is linear,

$$f(x, t) = A(t)x, \quad g(x, t) = C(t)x \quad (15)$$

the conditional mean and covariance can be generated from the finite-dimensional system:

$$d\hat{x}_t = A(t)\hat{x}_t dt + \Sigma(t)C'(t)W^{-1}(t)[dz - C(t)\hat{x}_t dt]; \quad x_0 = \bar{x}_0 \quad (16)$$

$$\dot{\Sigma}(t) = A(t)\Sigma(t) + \Sigma(t)A'(t) - \Sigma(t)C'(t)W^{-1}(t)C(t)\Sigma(t) + D(t)D'(t); \quad \Sigma(0) = \Sigma_0 \quad (17)$$

The filter defined by equations (16) and (17) is known as the Kalman (or Kalman-Bucy) filter. It can be derived from the general equations (13) and (14) in  $\hat{x}_t$  and  $\Sigma_t$ , from the Bucy representation (11), or directly using any of a variety of techniques such as the projection theorem. The simplification of equations (13) and (14) to (16) and (17) consists essentially of observing that when  $f$  and  $g$  are linear they commute with conditional expectation. Not only are no additional equations required beyond equations (16) and (17) to completely specify the conditional mean and covariance, but

equation (17) for the conditional covariance does not depend on the observation process  $z$ . Thus  $\Sigma$  is nonrandom and can be precomputed. To emphasize this, we write  $\Sigma(t)$  rather than  $\Sigma_t$  in this case.

The linearity of the system and the Gaussianness of the noise processes results in the state and observation processes being jointly Gaussian, so that the conditional density of the state  $x_t$  given the past observations is also Gaussian. This Gaussian posterior density is thus completely described by the conditional mean  $\hat{x}_t$  and the conditional covariance  $\Sigma_t$ . Furthermore, the conditional mean is a linear function of the data  $Z$  and, in fact, is the finite-dimensional linear function of the data given by equation (16). For the model equation (1a'') that includes a control causally derived from the observations with  $f(x,u,t) = A(t)x + B(t)u$  and  $g(x,t) = C(t)x$ , the state and observation processes are no longer Gaussian, but the conditional density of the state given past observations and controls is still Gaussian with conditional mean  $\hat{x}_t$ , the finite-dimensional linear function of past  $u$  and  $z$ , given by

$$d\hat{x}_t = A(t)\hat{x}_t dt + B(t)u_t dt + \Sigma(t)C'(t)W^{-1}(t)[dz_t - C(t)\hat{x}_t dt] \quad (18)$$

The conditional covariance is still given by equation (17) and remains precomputable and independent of  $u$  and  $z$  (this fact is of major importance in stochastic control situations).

Finally, we note the simple structure of the linear filter (18). As noted, for example, in reference 27, it consists of a model of the deterministic part of the system (1a'') (which generates, in particular, the first two terms on the right side of eq. (8)), a unity gain negative feedback loop that generates the innovations  $dz - C\hat{x}_t dt$ , and a gain  $\Sigma C'W^{-1}$  that operates on the innovations to produce the third term on the right side of equation (18). Imitation of this simple structure is one possible starting point in developing approximate filters for nonlinear problems.

## V. RELINEARIZED ESTIMATORS AND EXTENDED KALMAN FILTERS

Because linear estimation problems are so amenable to analysis and have finite-dimensional solutions, while nonlinear problems require infinite-dimensional filters, it is natural to attempt to approximate nonlinear problems by linear ones in the hope that the optimum filter for the linearized problem will suitably approximate the optimum nonlinear filter. In this vein, the most common approach is to continually relinearize the system equations (1) about the current state estimate  $\hat{x}_t$  using a Taylor series expansion in which second- and higher-order terms are neglected, namely,

$$f(x_t, t) = f(\hat{x}_t, t) + f_x(\hat{x}_t, t)[x_t - \hat{x}_t] \quad (19a)$$

$$g(x_t, t) = g(\hat{x}_t, t) + g_x(\hat{x}_t, t)[x_t - \hat{x}_t] \quad (19b)$$

where  $f_x(\hat{x}_t, t)$  denotes the partial derivative of  $f$  with respect to its first argument evaluated at  $\hat{x}_t$ . The system equations can then be written in the linear form

$$dx_t = f_x(\hat{x}_t, t)x_t dt + \bar{f}_t dt + D(t)dv_t \quad (20a)$$

$$dz_t = g_x(\hat{x}_t, t)x_t dt + \bar{g}_t dt + G(t)dw_t \quad (20b)$$

where the known (given  $\hat{x}_t$ ) quantities  $\bar{f}_t$  and  $\bar{g}_t$  are given by

$$\bar{f}_t = f(\hat{x}_t, t) - f_x(\hat{x}_t, t)\hat{x}_t, \quad \bar{g}_t = g(\hat{x}_t, t) - g_x(\hat{x}_t, t)\hat{x}_t$$

Then, with appropriate modifications to reflect that the linear system of interest is now equation (20), the Kalman filter (eqs. (16) and (17)) becomes, after collect-terms,

$$d\hat{x}_t = f(\hat{x}_t, t)dt + \Sigma_t g'_x(\hat{x}_t, t)W^{-1}(t)[dz_t - g(\hat{x}_t, t)dt] \quad (21)$$

$$\dot{\Sigma}_t = f_x(\hat{x}_t, t)\Sigma_t + \Sigma_t f'_x(\hat{x}_t, t) + D(t)D'(t) - \Sigma_t g'_x(\hat{x}_t, t)W^{-1}(t)g_x(\hat{x}_t, t)\Sigma_t \quad (22)$$

Note that the error covariance  $\Sigma_t$  now depends on the observation process via the dependence of  $f_x$  and  $g_x$  on  $\hat{x}$ . The suboptimum estimator (eqs. (21) and (22)) is usually called an extended Kalman filter.

Alternatively, instead of identifying the linearized system (20), one can simply substitute equation (19) into (13) and (14) for the mean and covariance of the nonlinear problem. The resulting equations can be simplified to equations (21) and (22), provided it is assumed that the suboptimum estimator  $\hat{x}$  satisfies  $E[x_t - \hat{x}_t | Z_t] = 0$  so that, for example, we have, from equation (19a),  $f_t \triangleq \widehat{f(x_t, t)} = f(\hat{x}_t, t)$ . By first identifying the linearized system (20), the need to make this explicit assumption is avoided. Yet another alternative is to take the structure of the optimum linear estimator as a starting point, substituting instead a model of the nonlinear system to obtain

$$d\hat{x}_t = f(\hat{x}_t, t)dt + L_t[dz_t - g(\hat{x}_t, t)dt] \quad (23)$$

where the gain  $L_t$  is yet to be determined. Subtracting equation (23) from (1), using the approximations (19), and writing  $\tilde{x}_t$  for  $x_t - \hat{x}_t$  yields

$$d\tilde{x}_t = f_x(\hat{x}_t, t)\tilde{x}_t dt + D(t)dv_t - L_t dw_t - L_t g_x(\hat{x}_t, t)\tilde{x}_t dt$$

and, at least for  $f_x$  and  $g_x$  regarded simply as time functions, the gain  $L_t$  that minimizes the covariance of  $\tilde{x}$  is identified from linear system theory as  $L_t = \Sigma_t g'_x(\hat{x}_t, t)W^{-1}(t)$ , thus again leading to the extended Kalman filter (eqs. (21) and (22)).

The degree to which the suboptimum estimate  $\hat{x}_t$  given by the extended Kalman filter approximates the optimum (conditional mean) is obviously related

to the extent to which the second- and higher-order terms in the Taylor series expansion are negligible. The quality of this latter approximation will be improved when the nonlinearities are small and when the deviations of the state and observations (from  $\hat{x}$  and  $\hat{z}$ ) are small, that is, when  $\Sigma_0$  and the noise covariances  $DD'$  and  $GG' = W$  are small. However, almost nothing is known analytically about the performance or quality of approximation of extended Kalman filters (exact evaluation requires an infinite-dimensional calculation) and performance characteristics as mean-square error, sensitivity, etc., are evaluated in specific cases by simulation and Monte Carlo techniques. These simulation studies are usually limited to comparisons of competing suboptimal designs, rather than relating the suboptimum estimator to the optimum. Bias errors ( $E\tilde{x} \neq 0$ ) and divergence problems are not infrequently encountered. Remedies for these tend to be limited to ad hoc rules of thumb (e.g., a trial-and-error increase of  $DD'$  in the simulations, which has the effect of giving more relative weight to the more recent observations).

Various alternatives to continual relinearization about the current state estimate are possible. One is to linearize about a prechosen nominal trajectory; this leads to a precomputable filter gain  $L_t$  in equation (23) and precomputable  $\Sigma$ , but performance remains difficult to assess, and can be expected to be worse than for continually relinearized estimators. A second possibility is to retain second-order (or even more) terms in the Taylor series expansion about  $\hat{x}$ , which on substitution into equations (13) and (14) and simplification yields estimators considerably more complicated than equations (21) and (22). Again, performance evaluation is limited to simulation and Monte Carlo techniques (ref. 28). Other possibilities include statistical linearization (refs. 3 and 13) and a wide-sense approach in which certain nonlinear problems are converted via transformation into a linear problem with non-Gaussian initial state (ref. 10). Like the extended Kalman filter (eqs. (21) and (22)), these estimators all suffer from the absence of analytical methods for design, performance evaluation, error analysis, sensitivity studies, etc., and these characteristics must be evaluated by simulation. Comparisons between competing design procedures are severely limited by the absence of any small collection of accepted benchmark problems that might form a standard of comparison in simulation studies. Finally, any comparisons are almost always between competing suboptimum estimators and not between suboptimum and optimum.

## VI. APPROXIMATIONS TO $p_t(x_t | Z_t)$

Another approach to approximate nonlinear filtering is to approximate the conditional density  $p_t(x_t | Z_t)$  in terms of a finite number of basis functions. Expansions used here include "point mass," Fourier, Hermite polynomials, spline functions, and sums of Gaussian densities (refs. 29-35). These are usually done for discrete-time problems and typically use a version of the Bayes rule derived from the Bucy representation theorem. Because the effects of truncation and other questions related to quality of approximation have not yielded to analytic treatment, Monte Carlo methods are used to assess filter performance, data-processing rates, the number of terms needed, sensitivity, etc. Different expansions yield different data-processing rates and different

estimation accuracies; what works well for one problem may not work well for another. Computational effort is typically large. Comparison studies have been limited to Monte Carlo comparisons between suboptimal estimators; almost nothing is known as to how the performance of the suboptimal filters compares to the optimum achievable performance.

## VII. PERFORMANCE BOUNDS

The infinite-dimensionality of the optimum nonlinear filter, on the one hand, and the difficulty in evaluating the performance of extended Kalman filters and other suboptimum estimators on the other makes it desirable to seek out an analytical framework within which suboptimum filters can be designed and their performance evaluated with realistic computational effort. One direction along these lines which has received attention is an analysis in terms of precomputable upper and lower bounds. To establish bounds, some explicit assumptions are needed on the functions  $f$  and  $g$  in equations (1a) or (1a'') and (1b), and much of the work here considers systems incrementally conic (or cone-bounded) in the sense that

$$f(x + \delta, u + \gamma, t) - f(x, u, t) = A(t)x + B(t)u + \epsilon_1(t) + \epsilon_2(t)$$

$$g(x + \delta, t) - g(x, t) = C(t)x + \epsilon_3(t)$$

$$\|\epsilon_1\| \leq a(t)\|\delta\|, \quad \|\epsilon_2\| \leq b(t)\|\gamma\|, \quad \|\epsilon_3\| \leq c(t)\|\delta\|$$

Thus incrementally conic systems are linear to within a uniformly Lipschitz residual. Cone parameters  $A$ ,  $B$ , and  $C$  define a nominal linear system, while  $a$ ,  $b$ , and  $c$  measure the degree of nonlinearity. Many common modeling functions are incrementally conic, including sinusoids, inverse tangents, and saturating nonlinearities; also included are linear systems whose parameters are known only to within specified tolerances. Polynomials and exponentials in  $x$  and  $u$  are not incrementally conic.

Within the framework of incrementally conic systems, upper bounds have been derived in reference 36 on the mean-square error associated with the parametrized (by  $K$ ) family of easily implemented estimators:

$$d\hat{x}_t = f(\hat{x}_t, u_t, t)dt + K(t)[dz_t - g(\hat{x}_t, t)dt] \quad (24)$$

These are matrix-ordering bounds of the form

$$E[(x_t - \hat{x}_t)(x_t - \hat{x}_t)'] \leq P_K(t) \quad (25)$$

where the  $n \times n$  matrix  $P_K(t)$  satisfies a linear differential equation. Thus, for any given  $K(t)$  in equation (24), the corresponding mean-square estimation error is guaranteed to be less than  $P_K(t)$ . Furthermore, there exists a choice  $K^*(t)$  of  $K$  such that the corresponding upper bound  $P^*(t) = P_{K^*}(t)$  is at all times smaller in the matrix ordering sense than that for any other  $K(t)$  (i.e.,  $P^*(t) \leq P_K(t)$  for all  $K$  and all  $t$ ). Because

$K^*(t)$  depends on the corresponding  $P^*(t)$ ,  $P^*$  satisfies a Riccati-like equation (just as the optimum gain for linear estimation problems depends on the corresponding error covariance  $\Sigma(t)$ , which results in  $\Sigma(t)$  satisfying a Riccati equation). This does not say that the actual mean-square error of the estimator (1) is smaller for  $K^*$  than for any other  $K$ . What it *does* say is that when  $K^*$  is used the *guaranteed* performance is smaller than the performance guaranteed using any other  $K$ .

Under certain additional technical assumptions, it is also possible to derive a lower bound on the expected error covariance that can be achieved by *any* causal filter, including the optimum. This matrix-ordering bound takes the form (ref. 37)

$$E\{\text{cov}[x_t | Z_t]\} \geq [1 - r(t)]\hat{P}(t) \quad (26)$$

where  $\hat{P}(t)$  is the optimum error covariance for the nominal linear estimation problem (and thus depends only on  $A$ ,  $C$ ,  $\Sigma_0$  and the noise covariances  $DD'$  and  $W$ ), while the "shrinkage" factor  $r(t) < 1$  depends on  $a$  and  $c$  as well.

Both upper and lower bounds share most of the properties of the optimum error covariance for linear problems: in particular, they are easily computed by contemporary standards, and they are control-law independent. The lower bound converges uniformly and with finite ultimate to the optimum error covariance  $\hat{P}$  of the nominal linear problem as the nonlinearity vanishes ( $a, c \rightarrow 0$ ); the upper bound converges similarly for reasonable choice of  $K$ , including the bound minimal gain  $K^*$  and the optimum gain for the nominal linear problem. Together, the upper and lower bounds define a range in which must lie both the true optimum performance and the actual mean-square error of the suboptimum estimator (eq. (24)). In some specific cases, this range may be sufficiently small that the suboptimum estimator can be deemed sufficiently close to optimum. In others, the guaranteed performance provided by the upper bound might meet acceptable performance specifications with the optimum achievable performance being irrelevant. In such cases, the bounds provide a simple analytical and computational framework for design and evaluation. If the guaranteed performance is inadequate (but still potentially achievable in view of the lower bound), one can then turn to alternative suboptimum designs such as those discussed earlier, with performance assessment by simulation and design modification by trial and error. Similar upper bounds can be found for extended Kalman filters (where  $K$  can depend on  $\hat{x}$  as well as  $t$ ), but these upper bounds are larger than those where  $K$  depends only on  $t$ . The upper and lower bounds can be used to derive performance bounds for stochastic control problems (ref. 38); they have also been extended to smoothing and prediction (ref. 12).

Other lower bounds of a Cramer-Rao type have been derived in references 39 to 41. Upper bounds for certain rather restricted classes of systems can be found in reference 42 and references cited in reference 36.

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A SURVEY OF DESIGN METHODS FOR FAILURE DETECTION  
IN DYNAMIC SYSTEMS\*

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SUMMARY

In this paper, we survey a number of methods for detecting abrupt changes (such as failures) in stochastic dynamical systems. We concentrate on the class of linear systems, but the basic concepts, if not the detailed analyses, carry over to other classes of systems. The methods surveyed range from the design of specific failure-sensitive filters, to the use of statistical tests on filter innovations, to the development of jump process formulations. Trade-offs in complexity versus performance are discussed.

I. INTRODUCTION

With the increasing availability and decreasing cost of digital hardware and software, there has developed a desire in several disciplines for the development of sophisticated digital system design techniques that can greatly improve overall system performance. A good example of this can be found in the field of digital aircraft control (see, e.g., refs. 1-3), where a great deal of effort is being put into the design of aircraft with reduced static stability, flexible wings, etc. Such vehicles can provide improved performance in terms of drag reduction and decreased fuel consumption, but they also require sophisticated control systems to deal with problems such as active control of unstable aircraft, suppression of flutter, detection of system failures, and management of system redundancy. The demands on such a control system are beyond the capabilities of conventional aircraft control-system design techniques, and the use of digital techniques is essential.

Another example can be found in the field of electrocardiography. In recent years, a great deal of effort has been devoted to the development of digital techniques for the automatic diagnosis of electrocardiograms (ECG) (see, e.g., ref. 4). Such systems can be for preliminary screening of large numbers of ECG's, for monitoring patients in a hospital, etc.

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In this paper, we review some of the recent work in one area of system theory that is important in both of these examples, as well as in many other system design problems. Specifically, we discuss the problem of detecting abrupt changes in dynamical systems. In the aircraft control problem, one is concerned with the detection of actuator and sensor failures, while in the ECG analysis problem, one wants to detect arrhythmias — sudden changes in the rhythm of the heart. For simplicity in our discussion, we refer to all such abrupt changes as "failures," although, as in the ECG example, the abrupt change need not be a physical failure. Our aim in this survey is to provide an overview of a number of the basic concepts in failure detection. The problem of system reorganization subsequent to the detection of a failure is considered in several of the references. We will point out these references in the sequel, but we will concentrate primarily on the detection problem.

The design of failure detection systems involves the consideration of several issues. One is usually interested in designing a system that will respond rapidly when a failure occurs; however, in high-performance systems, one often cannot tolerate significant degradation in performance during normal system operation. These two considerations are usually in conflict. That is, a system designed to respond quickly to certain abrupt changes must necessarily be sensitive to certain high-frequency effects, and this, in turn, will tend to increase the sensitivity of the system to noise (via the occurrence of false alarms signaled by the failure detection system). The tradeoff between these design issues is best studied in the context of a specific example in which the costs of the various tradeoffs can be assessed. For example, one might be more willing to tolerate false alarms in a highly redundant system configuration than in a system without substantial backup capabilities.

Generally, one would like to design a failure detection system that takes system redundancy into account. For example, in a system containing several backup subsystems, we may be able to devise a simple detection algorithm that is easily implemented but yields only moderate false-alarm rates. On the other hand, by implementing a more complex failure detection algorithm that takes careful account of system dynamics, one may be able to reduce requirements for costly hardware redundancy.

In addition to taking hardware issues into consideration, the designer of failure detection systems should consider the issue of computational complexity. One clearly needs a scheme that has reasonable storage and time requirements. It would also be useful to have a design methodology that admits a range of implementations, allowing a tradeoff study of system complexity versus performance. In addition, it would be desirable to have a design that takes advantage of new computer capabilities and structures (e.g., designs amenable to modular or parallel implementations).

In this paper, we survey a variety of failure detection methods and, keeping the issues mentioned above in mind, we will comment on the characteristics, advantages, disadvantages, and tradeoffs involved in the various techniques. To provide this survey with some organization and to point out

some of the key concepts in failure detection system design, we have defined several categories of failure detection systems and have placed the designs we have collected into these groups. Clearly, such a grouping can only be a rough approximation, and we caution the reader against drawing too much of an inference about individual designs based on our classification of them (several of the techniques could easily fall into a number of our classes). In addition, for brevity, we have limited our detailed discussions to only a few of the many techniques. Our choice of those techniques has been motivated by a desire to span the range of available methods and by our familiarity with certain of these algorithms. Finally, we have attempted to collect all of those studies of the failure detection problem of which we are aware, and we apologize for any oversights.

## II. FORMULATIONS OF FAILURE DETECTION PROBLEM

In this paper, we are mostly concerned with the analysis of linear stochastic models in the standard state space form:

System dynamics:

$$x(k+1) = \Phi(k)x(k) + B(k)u(k) + w(k) \quad (1)$$

Sensor equation:

$$z(k) = H(k)x(k) + J(k)u(k) + v(k) \quad (2)$$

where  $u$  is a known input and  $w$  and  $v$  are zero mean, independent, white Gaussian sequences with covariances defined by

$$E[w(k)w'(j)] = Q\delta_{kj}, \quad E[v(k)v'(j)] = R\delta_{kj} \quad (3)$$

where  $\delta_{kj}$  is the Kronecker delta. We think of equations (1) to (3) as describing the "normal operation" or "no-failure" model of the system of interest. If no failures occur, the optimal state estimator is given by the discrete Kalman filter equations (ref. 5):

$$\hat{x}(k+1|k) = \Phi(k)\hat{x}(k|k) + B(k)u(k) \quad (4)$$

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)\gamma(k) \quad (5)$$

$$\gamma(k) = z(k) - H(k)\hat{x}(k|k-1) - J(k)u(k) \quad (6)$$

where  $\gamma$  is the zero mean, Gaussian innovation process and gain  $K$  is calculated from

$$P(k+1|k) = \Phi(k)P(k|k)\Phi'(k) + Q \quad (7)$$

$$V(k) = H(k)P(k|k-1)H'(k) + R \quad (8)$$

$$K(k) = P(k|k-1)H'(k)V^{-1}(k) \quad (9)$$

$$P(k|k) = P(k|k-1) - K(k)H(k)P(k|k-1) \quad (10)$$

Here  $P(i|j)$  is the estimation error covariance of the estimate  $\hat{x}(i|j)$  and  $V(k)$  is the covariance of  $\gamma(k)$ . We refer to equations (4) to (10) as the "normal mode filter" in the sequel.

In addition to the above estimator, one may also have a closed-loop control law, such as the linear law,

$$u(k) = G(k)\hat{x}(k|k)$$

We then obtain the normal operation configuration depicted in figure 1.

The problem of failure detection is concerned with detecting abrupt changes in a system, as modeled by equations (1) to (3). Such abrupt changes can arise in a number of ways. For example, in aerospace applications, one is often concerned with the failure of control actuators and surfaces. Such abrupt changes can manifest themselves as shifts in the control gain matrix  $B$ , increased process noise, or as a bias in equation (1) (as might arise if a thruster developed a leak (ref. 6)). In addition, failures of sensors may take the form of abrupt changes in  $H$ , increases in measurement noise, or as biases in equation (2). For simplicity, we refer to abrupt changes in equation (1) as "actuator failures," and shifts in equation (2) are called "sensor failures." Again, we point out that, in many applications, shifts in equation (1) or (2) may be used to model changes in observed system behavior that have nothing to do with actuators or sensors.

The main task of a failure detection and compensation design is to modify the normal mode configuration to include the capability of detecting abrupt changes and compensating for them by activating backup systems, adjusting the feedback design appropriately, etc. Conceptually, we think of the detection-compensation system as part of the filtering portion of the feedback loop. As illustrated in figures 2 and 3, the resulting filter design can take one of two forms: either we perform a complete redesign of the filter, replacing equations (4) to (10) with a filter that is sensitive to failures or we design a system that monitors the normal system configuration and adjusts the system accordingly. We will discuss examples of both structures.

As mentioned earlier, we will concentrate primarily on the problem of failure detection, which we consider to consist of three tasks — alarm, isolation, and estimation. The alarm task simply consists of making a binary decision, either that something has gone wrong or that everything is fine. The problem of isolation is that of determining the source of the failure (e.g., which sensor or actuator has failed, what type of arrhythmia has occurred, etc.). Finally, the estimation problem involves the determination of the extent of failure. For example, a sensor may become completely non-operational (an "off" or "hard-over" failure), or it may simply suffer degradation in the form of a bias or increased inaccuracy, which may be modeled as



an increase in the sensor noise covariance. In the latter case, estimates of the bias or the increase in noise may allow continued use of the sensor, albeit in a degraded mode. Clearly, the extent to which we need to perform these various tasks depends on the application. If a human operator is available, we may only be interested in generating an alarm that tells him to perform further tests. In other systems in which backups are available, we might settle for failure isolation without estimation. On the other hand, in the absence of hardware redundancy, we may be interested in using a degraded instrument and thus would need estimation information.

Intuitively, we can associate increased software system complexity with the tasks, that is, isolation requires more sophisticated data processing than an alarm, and estimation more than isolation. On the other side, as we increase failure detection capabilities, we may be able to *decrease* hardware redundancy. Also, in some applications, we may be able to delay isolation and estimation until after an alarm has been sounded. In such a sequential structure, one increases detector complexity *after* a failure has been detected, thereby reducing the computational burden during normal operation. Again the details of such considerations depend on the particular application.

Another tradeoff involving failure detection system complexity involves its relation to detection system performance. For example, one might expect that one could achieve better alarm performance by using *a priori* knowledge concerning likely failure modes. That is, by looking for specific forms of system behavior characteristic of certain failures, one should be able to improve detection performance. Thus it seems likely that alarm performance (as measured by the tradeoff between false alarms and missed detections) will be improved if we attempt simultaneous detection, isolation, and estimation of failures. This tradeoff of complexity versus performance is extremely important in the design of failure detection systems.

In the following sections, we discuss several failure detection methods and comment on their characteristics with respect to the issues mentioned in this and the preceding section. We have not provided a general set of failure models to be considered, as the various techniques are based on quite different failure models. These are described as we discuss the various methodologies.

### III. "FAILURE-SENSITIVE" FILTERS

Our first class of failure detection concepts is aimed at overcoming the problem of an "oblivious filter." As has been noted by many authors (refs. 5 and 7-9), the optimal filter defined by equations (4) to (10) performs well if there are no modeling errors. However, it is possible for the filter estimate to diverge if there are substantial unmodeled phenomena. The problem occurs because the filter "learns the state too well," that is, the precomputed error covariance  $P$  and filter gain  $K$  become small, and the filter relies on old measurements for its estimates and is oblivious to new measurements. Thus, if an abrupt change occurs, the filter will respond

quite sluggishly, yielding poor performance. Consequently, one would like to devise filter designs that remain sensitive to new data so that abrupt changes will be reflected in the filter behavior.

Two well-known techniques for keeping the filter sensitive to new data are the exponentially age-weighted filter studied by Fagin (ref. 7) and by Tarn and Zaborszky (ref. 8) and the limited memory filter proposed by Jazwinski (ref. 9). Others, such as increasing noise covariances or simply fixing the filter gain, are discussed by Jazwinski (ref. 5). These techniques yield only indirect failure information; that is, if an abrupt change occurs, these filters will respond faster than the normal filter, and one can base a failure detection decision on sudden changes in  $\hat{x}$ .

It is important to note a performance tradeoff evident in this method. As we increase our sensitivity to new data (by effectively increasing the bandwidth of the Kalman filter), our system becomes more sensitive to sensor noise, and the performance of the filter under no-failure conditions degrades. In some cases, this can be rather severe, and one may not be able to tolerate the degradation in overall system performance under no-failure conditions. One might then consider a two filter system — the normal mode filter (eqs. (4)-(10)) as the primary filter, with this type of failure-sensitive filter as an auxiliary monitor, used only to detect abrupt changes. We remark that the tradeoff between detection performance and filter behavior under normal conditions is a characteristic of all failure detection systems and is analogous to the costs associated with false alarms and missed detections in standard detection problems (ref. 10).

The techniques mentioned so far are rather indirect failure detection approaches. Several methods have been developed for the design of filters sensitive to specific failures. One method involves the inclusion of several "failure states" in the dynamic model (eqs. (1)-(3)). Kerr (ref. 11) has considered a procedure in which failure modes, such as the onset of biases, are included as state variables. If the estimates of these variables vary markedly from their nominal values, a failure is declared. A two-confidence interval overlap decision rule for failure detection using such failure states is described and its performance is analyzed in reference 11. Note that this approach provides failure isolation and estimation at the expense of increased dimensionality and some performance degradation under no-failure conditions (inclusion of the added states effectively opens up the bandwidth of the Kalman filter).

An alternative to the addition of failure states to the dynamic model is the class of detector filters developed by Beard (ref. 12) and Jones (ref. 13). Their work has led to a systematic design procedure for detecting a wide variety of abrupt changes in linear time-invariant systems. They consider the continuous-time, time-invariant, deterministic system model:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (11)$$

$$z(t) = Cx(t) \quad (12)$$

and design a filter of the form

$$\frac{d}{dt} \hat{x}(t) = A\hat{x}(t) + D[z(t) - C\hat{x}(t)] + Bu(t) \quad (13)$$

The primary criterion in the choice of the gain matrix  $D$  is not that equation (13) provide a good estimate of  $x$  (as it is with observers or optimal estimators), but rather that the effects of certain failures are accentuated in the filter residual:

$$\gamma(t) = z(t) - C\hat{x}(t) \quad (14)$$

The basic idea is to choose  $D$  so that particular failure modes manifest themselves as residuals that remain in a fixed direction or in a fixed plane.

To illustrate the Beard-Jones approach, consider a simple example from reference 12. Suppose we wish to detect a failure of the  $i$ th actuator (i.e., in the actuator driven by the  $i$ th component of  $u$ ). If we assume the failure takes the form of a constant bias, our state equation becomes

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B[u(t) + ve_i] \\ &= Ax(t) + Bu(t) + vb_i, \quad t \geq t_0 \end{aligned} \quad (15)$$

where  $e_i$  is the  $i$ th standard basis vector,  $b_i$  is the  $i$ th column of  $B$ , and  $t_0$  is the (unknown) time of failure. Suppose we consider the case of full state measurement — let  $C = I$ . In this case, we obtain a differential equation for the residual

$$\dot{\gamma}(t) = [A - D]\gamma(t) + vb_i \quad (16)$$

If we choose  $D = \sigma I + A$ , we obtain

$$\begin{aligned} \dot{\gamma}(t) &= -\sigma\gamma(t) + vb_i \\ \gamma(t) &= e^{-\sigma(t-t_0)} \gamma(t_0) + \frac{v[1 - e^{-\sigma t}]}{\sigma} b_i \end{aligned} \quad (17)$$

Thus, as the effect of the initial condition dies out,  $\gamma(t)$  maintains a fixed direction ( $b_i$ ) with magnitude proportional to failure size ( $v$ ). Note that as we increase  $\sigma$  (thus increasing filter gain), the initial condition dies out faster, but the magnitude of the steady-state value of  $\gamma$  decreases. Thus, if there is any noise in the system, we cannot make  $\sigma$  arbitrarily large.

In their work, Beard and Jones consider the design of such filters for an extremely wide variety of failure modes, including actuator and sensor shifts and shifts in  $A$  and  $B$ . The initial deterministic analysis for all of these cases was considered by Beard (ref. 12), while a systematic design procedure is given by Jones (ref. 13) for the design of gain  $D$  to allow

detection of several failure modes. Jones' approach is quite geometric in nature, and his formulation allows one to gain considerable insight into the detection problem. As pointed out in reference 13, the gain selection problem is quite similar to the output decoupling problem and requires the introduction of the important concept of "mutually detectable failure modes" to answer the question of whether one can simultaneously distinguish between several types of failures. Thus the question of failure isolation is of central importance in the design methodology derived in reference 13.

The results in references 12 and 13 represent perhaps the most thorough study of the basic concepts underlying failure detection. The tradeoff between detection and filter performance is discussed in depth in reference 13 and an attempt is made in reference 12 to introduce the concept of the level of redundancy in a dynamical system.

As mentioned in the example, the basic design procedure is deterministic. However, in this simple example, we can see how one can take noise into account. If the system (eqs. (11) and (12)) contains noise, we have seen that one may not wish to make the scalar  $\sigma$  as large as possible. In fact, one could choose  $\sigma$  so as to minimize the mean-square estimation error in the detector filter when there is no failure. In his thesis, Jones (ref. 13) describes a procedure in which one first chooses the structure of  $D$  for failure detection purposes and then chooses the remaining free parameters to minimize the estimation error covariance. Although this yields a suboptimal filter design, it may work quite well, as it did in the problem reported in reference 13.

The Jones-Beard design methodology is extremely useful conceptually, it can be used to detect a wide variety of failures, and it provides detailed failure isolation information. It is suboptimal as an estimator, and if this presents a serious problem, one might wish to use the detector filter as an auxiliary monitoring system. This appears to be only a minor drawback, and the major limitation of the approach is its applicability only to time-invariant systems.

#### IV. VOTING SYSTEMS

Voting techniques are often useful in systems that possess a high degree of parallel hardware redundancy. Memoryless voting methods can work quite well for the detection of "hard" or large failures, and Gilmore and McKern (ref. 14), Pejsa (ref. 15), and Ephgrave (ref. 16) discuss the successful application of voting techniques to the detection of hard gyro failures in inertial navigation systems.

In standard voting schemes, one has (at least) three identical instruments. Simple logic is then developed to detect failures and eliminate faulty instruments, for example, if one of the three redundant signals differs markedly from the other two, the differing signal is eliminated. Recently, Broen (ref. 17) has developed a class of voter estimators that

possesses advantages relative to standard voting techniques. Consider the dynamical system

$$x(k+1) = \Phi x(k) \quad (18)$$

with a triply redundant set of sensors:

$$\left. \begin{aligned} y_1(k) &= H_1 x(k) + v_1(k) \\ y_2(k) &= H_2 x(k) + v_2(k) \\ y_3(k) &= H_3 x(k) + v_3(k) \end{aligned} \right\} \quad (19)$$

Broen develops a set of recursive filter equations for computing the estimate  $\hat{x}(k)$  that minimizes

$$J_k = \sum_{i=0}^k \sum_{j=1}^3 w_{ji} \gamma_j'(i) R_j^{-1} \gamma_j(i) \quad (20)$$

where  $R_j$  is the covariance of the measurement noise  $v_j$  and  $\gamma_j$  is the innovations sequence

$$\gamma_j(i) = y_j(i) - H_j \Phi^{i-k} \hat{x}(k) \quad (21)$$

Here  $w_{ji}$  is a function of  $y_1(i)$ ,  $y_2(i)$ , and  $y_3(i)$  which is large if  $y_j(i)$  is close to the other two  $y_m(i)$  terms and is small if  $y_j(i)$  deviates greatly from the other two. In this way, one obtains a "soft" voting procedure in which faulty sensors are smoothly removed from consideration. This greatly alleviates the cost of false alarms, but the price is the on-line computation of the filter gain (which is a function of the  $w_{ji}$ ). Note that in equation (19), Broen appears to allow the  $y_i$  to be physically different sensors (different  $H_i$  terms), but the analysis of his paper makes it clear that he requires identical sensors (i.e.,  $H_1 = H_2 = H_3$ ).

Voting schemes are generally relatively easy to implement and usually provide fast detection of hard failures, but they are applicable only in systems that possess a high level of parallel redundancy. They do not generally take advantage of redundant information provided by unlike sensors, and thus cannot detect failures in single or even doubly-redundant sensors. In addition, voting techniques can have difficulties in detecting "soft" failures (such as a small bias shift).

## V. MULTIPLE HYPOTHESIS FILTER DETECTORS

A rather large class of adaptive estimation and failure detection schemes involves the use of a "bank" of linear filters based on different hypotheses concerning the underlying system behavior. In the work of Athans and Willner

(ref. 18) and Lainiotis (ref. 19), several different sets of system matrices are hypothesized. Filters for each model are constructed, and the innovations from the various filters are used to compute the conditional probability that each system model is the correct one. In this manner, one can do simultaneous system identification and state estimation. In addition, an abrupt change in the probabilities can be used to detect changes in true system behavior. This technique has been investigated in the context of the adaptive control of the F-8C digital fly-by-wire aircraft by Athans *et al.* (ref. 20) and also has been applied to the problem of classifying rhythms and detecting rhythm shifts in electrocardiograms. Extremely good results in the latter case are reported by Gustafson *et al.* (ref. 21).

Techniques involving multiple hypotheses have also been used to design failure detection systems. Montgomery *et al.* (refs. 22 and 23) have used such a technique for digital flight-control systems and have studied its robustness in the presence of nonlinearities via simulations. Recently, a technique involving a bank of observers has been devised (ref. 24), and a successful application to a hydrofoil sensor failure problem is reported by Clark *et al.* (ref. 24). Also, Willsky *et al.* (refs. 25 and 26) have applied the methodology devised by Buxbaum and Haddad (ref. 27) to study failure detection for an inertial navigation problem. We will briefly describe this technique to illustrate some of the concepts underlying the bank-of-filters approach. We also refer the reader to Wernersson (ref. 28) for a technique similar to that discussed in reference 26.

Consider the system

$$x(k+1) = \Phi(k)x(k) + w(k) \quad (22)$$

$$z(k) = H(k)x(k) + v(k) \quad (23)$$

We are interested in detecting sudden shifts in certain of the components of  $x$  (e.g., bias states). We model these shifts by choosing the distribution of  $w$  appropriately. Let  $\{f_1, \dots, f_r\}$  be the set of hypothesized failure directions. We then assume that  $w$  has a high probability of being the usual process noise and a small probability of including a burst of noise in each of the failure directions. Thus the density for  $w(k)$  is

$$p_0 N(0, Q) + \sum_{i=1}^r p_i N(0, Q + \sigma_i f_i f_i') \quad (24)$$

$$\sum_{i=0}^r p_i = 1, \quad p_0 \gg p_i \quad i = 1, \dots, r \quad (25)$$

Here  $N(m, P)$  is a normal density with mean  $m$  and covariance  $P$ .

If we hypothesize such a density at each point in time and if we assume that  $x(0)$  is normally distributed, we have the following expression for the conditional density of  $x(k)$ , given  $z(1), \dots, z(k)$ :

$$p(x, k) = \sum_{i_0=0}^r \dots \sum_{i_{k-1}=0}^r p_{\underline{i}} N(\eta_{\underline{i}}, P_{\underline{i}}) \quad (26)$$

Here  $\underline{i} = (i_0, \dots, i_{k-1})$  and the density has the following interpretation. Let  $\underline{j} = (j_0, \dots, j_{k-1})$  be a random  $k$ -tuple where  $j_s = i$  if there is a shift in the  $f_i$  direction at time  $s$  ( $i = 0$  is used to denote no shift). Then

$$p_{\underline{i}} = \Pr[\underline{j} = \underline{i} | z(1), \dots, z(k)] \quad (27)$$

and  $\eta_{\underline{i}}$  and  $P_{\underline{i}}$  are the mean and covariance of the Kalman filter designed assuming  $\underline{j} = \underline{i}$  (i.e., assuming  $w(s)$  has covariance  $Q + \sigma_i f_{i_s} f_{i_s}'$ ). The  $P_{\underline{i}}$  terms can be computed sequentially as a function of the various filter innovations. We refer the reader to references 25 to 27 for the details of the calculations.

Note that the implementation of equation (26) requires an exponentially growing bank of filters (there are  $(r+1)^k$  terms in eq. (26)). To avoid this problem, a number of approximation techniques have been proposed (refs. 25-27). The one used in reference 26 involves hypothesizing shifts only once every  $N$  steps. At the end of each  $N$  step period, we "fuse" the  $(r+1)$  densities into a single density and begin the procedure again. In this way, we implement only  $(r+1)$  filters at any time. Note that the techniques devised in references 18, 19, and 22 do not involve growing banks of filters (as the number of hypothesized models do not grow in time). However, it is possible for all filters in the bank to become oblivious, and thus shifts between the hypotheses may go undetected (see refs. 21 and 26 for examples). The technique of periodic fusing of the densities and initiation of new banks effectively avoids this problem (as would designing the original bank using age-weighted filtering techniques).

The technique described above was applied to the problem of detecting gyro and accelerometer bias shifts in a time-varying inertial calibration and alignment system. The results of these tests are extremely impressive. This is not surprising as the multiple-hypothesis method computes precisely the quantities of interest — the probabilities of all types of failures under consideration. The cost associated with such a high level of performance is an extremely complex failure detection system. Note, however, that the parallel structure of the system allows one to consider highly efficient parallel processing computer implementations. In addition, the use of reduced-order filters for the various failure hypotheses may increase the practicality of such a scheme, or one might consider the use of a simpler detection-only system to detect failures, with a switch to a multiple hypothesis procedure for failure isolation and estimation after a failure has been detected.

However, even if such a failure detection scheme cannot be implemented in a particular application, it provides a useful benchmark for comparison

with simpler techniques. In addition, by studying the simulation of a multiple hypothesis method, one can gain useful insight into the dynamics of failure propagation and detection (see the discussion in ref. 26).

McGarty (ref. 29) has developed a method for rejecting bad measurements which bears some similarity to the approach just discussed. Each measurement has a binary random variable  $g(k)$  associated with it. If  $g(k) = 1$ , the measurement is "good" (i.e., the measurement contains the signal of interest), while  $g(k) = 0$  denotes a bad data point (measurement is pure noise). McGarty devises a maximum likelihood approach for estimating the values of the exponentially growing set of possibilities  $[g(i) = 1 \text{ or } 0, i = 1, \dots, k]$ . He also allows these variables to have a sequential correlation (i.e., knowing that the present measurement is good or bad says something about the next observation). A computationally feasible approximation method is devised and simulation results are described (see ref. 29 for details).

Recently, Athans *et al.* (ref. 30) also considered the problem of designing an estimator that can detect and remove bad or false measurements. Their approach is Bayesian in nature, that is, an estimate is generated of the *a posteriori* probability that a given measurement is false. The method of calculation of these pseudo-probabilities is quite similar to that used in the other multiple hypothesis methods (see refs. 18, 19, 22, 23, and 27). (The reader is referred to ref. 30 for details of the analysis and for a discussion of some successful simulation results.)

## VI. JUMP PROCESS FORMULATIONS

The problem of detecting abrupt changes in dynamical systems suggests the use of jump process techniques in devising system design methodologies (see refs. 31-33 for general results on jump processes). One models potential failures as jumps, characterized by *a priori* distributions that reflect initial information concerning failure rates. The size of the possible failures are usually taken to be known. One could, however, model failure magnitude as a random variable. This leads to a compound jump process formulation that greatly complicates the desired analysis. In any event, taking such a jump process formulation, one can devise failure-sensitive control laws and methods for computing the conditional probability of failure. Control problems of this type have received a great deal of attention in the literature. Swarder and Robinson (refs. 34-38) and Ratner and Luenberger (ref. 39) have considered the design of control laws which consider the possibility of sudden shifts in system matrices. The results they obtained are for the full-state feedback problem with no system randomness other than the jumping of the system matrices among a finite set of possible matrices.

Davis (ref. 40) has used nonlinear estimation techniques to solve a fault detection problem. His formulation is as follows: consider the scalar stochastic equations,



$$dx(t) = a(t)x(t)dt + g(t)dv(t) \quad (28)$$

$$dy(t) = h(t)x(t)dt + dw(t) \quad (29)$$

where  $w$  and  $v$  are independent Brownian motion processes and

$$a(t) = a_0(t)[1 - \xi(t)] + a_1(t)\xi(t) \quad (30)$$

where

$$\xi(t) = \left\{ \begin{array}{ll} 0, & t < T \\ 1, & t \geq T \end{array} \right\} \quad (31)$$

and  $T$  is a random variable. Here we interpret  $a_0$  as the unfailed dynamics, and  $a_1$  represents the failure mode. Davis derives the optimal, infinite-dimensional equations for the computation of the conditional mean of  $x$  and the conditional probability

$$\hat{\xi}(t|t) = \Pr[t \geq T | y(s), 0 \leq s \leq t] \quad (32)$$

An implementable approximation is described in reference 40, but evaluation of its performance has not yet been made.

Note that Davis' method leads to an estimate of  $x$  that is suboptimal under no-failure conditions. Chien (ref. 41) has devised a jump process formulation that avoids this difficulty for the problem of detecting a jump or a ramp in a gyro bias. He considers the dynamical model:

$$\dot{x}(t) = \omega x(t) + w(t) \quad (33)$$

where  $w$  is a white noise process. Three hypotheses are conjectured for the form of the gyro output:

Normal mode  $H_0$ :

$$z(t) = x(t) + v(t), \quad \forall t \quad (34)$$

Bias mode  $H_1$ :

$$z(t) = x(t) + m\xi(t) + v(t), \quad t > T \quad (35)$$

Ramp mode  $H_2$ :

$$z(t) = x(t) + n(t - T)\xi(t) + v(t), \quad t > T \quad (36)$$

where  $n$  and  $m$  are unknown constants,  $v$  is white noise,  $T$  is the time of failure, and  $\xi(t)$  is as in equation (31).

Chien's approach is as follows: design a filter based on  $H_0$  (which will thus yield the optimal estimate for  $t < T$ , assuming no false alarms occur) and determine the *steady-state* effect of the degradations  $H_1$  and  $H_2$  on the filter residuals. If one then hypothesizes a failure rate  $q$ , that is,

$$P(T > t) = e^{-qt} \quad (37)$$

and if one further assumes a nominal size for the bias  $m$ , one can then compute an approximate stochastic differential equation for  $\Pr(H_1 | z(s), s \leq t)$ , in which the input to this equation is the residual  $\gamma$  of the  $H_0$  filter. The details of the analysis are described in reference 41.

For his problem, Chien is able to demonstrate that his detection procedure — based on the assumption of a nominal value for the bias failure  $m$  — has the capability of detecting biases larger than  $m$  and also can be used to detect ramps (mode  $H_2$ ). Of course, the delay times until detection in these cases are greater than if one implemented a filter based on the proper bias size or if one were looking for a ramp (indicating the potential usefulness of estimating the failure magnitude). The major advantages of Chien's approach are the simplicity of the detector (implementation of a scalar stochastic equation) and the fact that one obtains an estimate of precisely the quantity of interest — the conditional probability of failure. The simplicity of the scheme may, in fact, make it a great deal more robust in the face of system modeling errors (such as the use of an extremely simplified gyro error model) than more sophisticated approaches. Also, this approach leads to no degradation in performance prior to detection of the failure. In addition, the use of a probabilistic description of the time of failure allows one to avoid the problem of the oblivious filter — that is, the fact that a failure can occur at any time has been incorporated in the design, which therefore will remain sensitive to new data.

The drawbacks of the scheme are the use of a fixed bias size and the use of the steady-state effect of the failure on the filter residual. The first of these may not be too much of a problem (as Chien pointed out), but the second may cause difficulties. Specifically, this limits the approach to time-invariant systems and filters. In addition, as the transient effect of the failure has been ignored, it may be difficult to make quick detections of certain changes (i.e., we may have to wait until the transient dies out). In the next section, we discuss an approach (the GLR method) that has several concepts in common with Chien's approach and that allows one to overcome these two drawbacks (at the cost of added computational complexity, of course).

Jump process formulations appear to be quite natural for failure detection problems. One usually makes approximations in the analysis to obtain implementable solutions. These simplifications impose some limitations on the capabilities of the designs, but presently there is no systematic analytical procedure for evaluating these limitations or for studying tradeoffs between design complexity and system performance.

## VII. INNOVATIONS-BASED DETECTION SYSTEMS

Chien's failure detection technique can also be placed in the class of failure detection methods that involve monitoring the innovations of a filter based on the hypothesis of normal system operation. In such a configuration, the overall system uses the normal filter until the innovation monitoring system detects some form of aberrant behavior. The fact that the monitoring system can be attached to a filter-controller feedback system is particularly appealing since overall system behavior is not disturbed until after the monitor signals a failure and since the monitoring system can be designed to be added to an existing system.

Mehra and Peschon (ref. 42) suggested a number of possible statistical tests to be performed on the innovations. One of these is a chi-squared test that was applied by Willsky *et al.* (refs. 25 and 26). Let  $\gamma(k)$  be the  $p$ -dimensional innovations for the filter defined by equations (4) to (10). If the system is operating normally, the innovations is zero mean and white with known covariance  $V(k)$ . In this case, the quantity

$$\ell(k) = \sum_{j=k-N+1}^k \gamma'(j)V^{-1}(j)\gamma(j) \quad (38)$$

is a chi-squared random variable with  $Np$  degrees of freedom (refs. 25, 26, and 42). If a system abnormality occurs, the statistics of  $\gamma$  change, and one can consider a detection rule of the form

$$\begin{array}{ccc} & \text{Failure} & \\ \ell(k) & > \epsilon & \\ & \text{No failure} & \end{array} \quad (39)$$

With the aid of chi-squared tables, one can compute the probability  $P_F$  of false alarm as a function of the innovations window length  $N$  and the decision threshold  $\epsilon$ . The probability  $P_D$  of correct detection depends on the particular failure mode (see ref. 26 and the discussion of the GLR approach to follow). We note that, for a given failure mode, as  $N$  increases the probability of correct detection may decrease — that is, by averaging a larger number of residuals we smooth out the effect of a failure on  $\gamma$ , and the detector may become somewhat oblivious (or at the very best respond quite slowly) to new data. On the other hand, too small a value of  $N$  may yield an unacceptably high value of  $P_F$ .

The implementation of the chi-squared test (eqs. (38) and (39)) is quite simple, but, as one might expect, one pays for this simplicity with rather severe limitations on performance. As described in references 25 and 26, this method was applied to the same inertial calibration and alignment problem to which the Buxbaum-Haddad multiple hypothesis approach (refs. 25-27), described in section V, was applied. The performance of the chi-squared test was mixed.

The method is basically an alarm method, that is, the system (eqs. (38) and (39)) makes no attempt to isolate failures, and one finds that those failure modes that have dramatic effects on  $\gamma$  are detectable by this method. However, more subtle failures are more difficult to detect with this simple scheme. Comparing the performance of the multiple hypothesis and chi-squared systems, we see that in some cases we can obtain superior alarm capabilities if we simultaneously attempt to do failure isolation and estimation. One can obtain some failure isolation information by considering the components of  $\gamma$  separately (this may be especially useful for sensor failures), and we refer the reader to references 25 and 26 for a detailed discussion of this and other aspects of the chi-squared method.

Another innovation-based approach, developed by Merrill (ref. 43), is motivated by a desire to suppress bad sensor data. Merrill devises a modification of the least-squares criterion to suppress extremely large residuals (which are given a very large weighting in the usual least-squares framework), and he applies his methodology to a power system application.

A final technique in this category has been studied by several researchers (refs. 6 and 44-48); we will describe the most general formulation of the approach, developed in references 44 and 45. This technique, which we call the generalized likelihood ratio (GLR) approach, was in part motivated by the shortcomings of the simpler chi-squared procedure. The GLR approach, which can be applied to a wide range of actuator and sensor failures, attempts to isolate different failures by using knowledge of the different effects such failures have on the system innovations. The method provides an optimum decision rule for failure detection and provides useful failure identification information for use in system reorganization subsequent to the detection of a failure. In addition, one can devise a number of simplifications of the technique and can study analytically the tradeoff between GLR complexity and GLR performance.

Consider the basic dynamical model (eqs. (1) to (3)). The following are four possible modifications of these equations that incorporate certain sudden system changes (see refs. 21, 44, and 45 for the physical motivation for these and other failure modes of the same general type):

Dynamic jump:

$$x(k+1) = \Phi(k)x(k) + B(k)u(k) + w(k) + v\delta_{k+1,\theta} \quad (40)$$

Here  $v$  is an unknown  $n$ -vector,  $\theta$  is the unknown time of failure, and  $\delta_{ij}$  is the Kronecker delta. Such a model can be used to model sudden shifts in bias states (as in the inertial problem studied in refs. 25 and 26).

Dynamic step:

$$x(k+1) = \Phi(k)x(k) + B(k)u(k) + w(k) + v\sigma_{k+1,\theta} \quad (41)$$

Here  $\sigma_{ij}$  is the unit step

$$\sigma_{ij} = \left\{ \begin{array}{ll} 1, & i \geq j \\ 0, & i < j \end{array} \right\} \quad (42)$$

This model can be used to model certain actuator failures (compare to the Beard-Jones example in section III; see eq. (15)).

Sensor jump:

$$z(k) = Hx(k) + Ju(k) + v(k) + v\delta_{k,\theta} \quad (43)$$

We can use this to model bad data points.

Sensor step:

$$z(k) = Hx(k) + Ju(k) + v(k) + v\sigma_{k,\theta} \quad (44)$$

Sudden changes in sensor biases fit into this model.

By the linearity of the system (eqs. (1) to (3)) and the filter (eqs. (4) to (10)), one can determine the effect of each failure mode on the innovations. The general form is

$$\gamma(k) = G(k;\theta)v + \tilde{\gamma}(k) \quad (45)$$

where  $\tilde{\gamma}(k)$  is the filter innovations if no failure occurs, and the matrix  $G$  can be precomputed (see refs. 45 and 48). This matrix, which is different for each of the four cases (eqs. (40) to (44)), is called the *failure signature matrix* and provides us with an explicit description of how various failures propagate through the system and filter.

The full-blown GLR method involves the following: we assume we are looking for one of the four classes of failures and have computed the appropriate signature matrix. Given the residuals, we compute the maximum likelihood estimates of  $v$  and  $\theta$  and, assuming that these estimates are correct, we compute the log-likelihood ratio for failure versus no failure (see ref. 10 for a general discussion of GLR methods). The implementation of the full GLR requires a linearly growing bank of matched filters, computing the best estimates of  $v$  assuming a particular value of  $\theta \in \{1, \dots, k\}$ .

A number of remarks can be made concerning the GLR system. We note that, as with other methods such as those of Buxbaum-Haddad or Chien, the inclusion of variable  $\theta$  to indicate our uncertainty as to the time of failure keeps the detection system sensitive to new data. However, it is the estimation of  $\theta$  that causes the growing complexity problem. On the other hand, even if the full GLR is not implementable, it can serve as a benchmark for other schemes and can, in fact, be used as a starting point for the design of simpler systems. One simplification that eliminates the growing complexity is the restriction of the estimate of  $\theta$  to a window

$$k - N \leq \theta \leq k - M \quad (45)$$

where the lower bound is included to limit complexity and the upper bound is set by failure observability and false-alarm considerations. Successful simulation runs with  $N = M$  (i.e., when we do not optimize  $\theta$  at all and have only one matched filter for  $\hat{v}$ ) are reported in reference 45. We remark only that the price one pays for "windowing" the estimate of  $\theta$  is a reduction in the accuracy of the estimate of  $v$ . For example, for  $N = M$ , we often are able to detect failures extremely quickly, but if  $\hat{\theta} = k - N$  is not the correct time of failure, the estimate of  $v$  may be severely degraded (e.g., our estimate of the slope of a ramp changes as we change our estimate of the time at which it started). We note that the estimation of  $\theta$  is similar to time-of-arrival estimation problems that arise in various applications (see ref. 49 for a general discussion of several techniques).

Note that even if the physical system and filter are time-invariant, the GLR monitoring system is time-varying, as the failure signature  $G$  includes transient effects. In some cases, one may be able to neglect these and use a simpler steady-state signature. This is quite similar to Chien's use (ref. 41) of the steady-state effect of the failure on the residuals, and the criticisms of that approach (given in section VI) apply here as well.

One can also simplify the implementation by either partially or completely specifying the failure magnitude  $v$ . Constrained GLR (CGLR) is based on the assumption that

$$v = \alpha f_1 \quad (46)$$

where  $\alpha$  is an unknown scalar and  $f_1$  is one of  $r$  possible failure directions. This technique is described in reference 45. If we completely specify  $v$ :

$$v = v_0 \quad (47)$$

we obtain the simplified GLR (SGLR) algorithm that is extremely simple to implement, as we have completely eliminated the need for the matched filters to estimate  $v$ . The use of specified failure sizes is similar to that proposed by Chien (ref. 41), although in SGLR one can use the time-varying failure signature, which should aid in failure detection. As initial results for the detection of electrocardiogram arrhythmias indicate (ref. 21), the estimation of  $v$  is not nearly as important for detection as matching the failure signatures. Also, by the use of several values of  $v_0$  (i.e., by implementing several parallel SGLRs), one can achieve a high level of failure isolation without a great deal of additional software complexity. In addition, one could consider a "dual-mode" procedure in which SGLR is used for alarm and isolation, with full GLR used only afterward to estimate the magnitude of the failure.

The various simplifications of GLR, as well as full GLR, are amenable to certain analysis, such as the calculation of  $P_F$ ,  $P_D$ , and (at least for SGLR) the expected time delay in detection. By performing such analyses, one can study in detail the tradeoff between complexity and performance. A methodology for such comparisons is presently being developed and is being applied to an aircraft failure detection problem. Initial results are reported by Chow *et al.* (ref. 48), and a description of a detailed methodology will be reported in the near future (see ref. 50). In addition to the calculation of  $P_F$  and  $P_D$ , the comparison methodology reported in reference 50 includes the computation of cross-detection probabilities — that is, the probability of detecting a failure of type A when a failure of type B has occurred. Such information can be useful in designing failure isolation procedures and also in determining whether failure detector A can be successfully used as an alarm for failures of type B. This can lead to substantial simplifications in a failure alarm system. (See refs. 21, 45, and 48 for successful simulations of the GLR method.)

Presently, the GLR method is being extended to other failure modes: hard-over actuator, increased process noise, hard-over sensor, and increased sensor noise.

Hard-over actuator failure:

$$x(k+1) = \phi(k)x(k) + [B + M\sigma_{k+1,0}]u(k) + w(k) \quad (48)$$

With this model, we can take into account complete (or "off") failures of certain actuators. For example, an off failure of the  $i$ th actuator can be modeled by choosing  $M$  all zero except for the  $i$ th column, which is taken to be the negative of the  $i$ th column of  $B$ . The GLR detector for equation (48) is presently under development (refs. 48 and 50), and we note that this model is more difficult than the others as the effect of the failure is modulated by the input values  $u(k)$ .

Increased process noise failure:

$$x(k+1) = \phi(k)x(k) + B(k)u(k) + w(k) + \xi(k)_{k+1,0} \quad (49)$$

Here  $\xi$  is additional white process noise.

Hard-over sensor failure:

$$z(k) = Hx(k) + Ju(k) + v(k) + [Mx(k) + Su(k)]\sigma_{k,0} \quad (50)$$

Here the failures are modulated by  $u$  and  $x$ , and a failure of the  $i$ th sensor is modeled by choosing the  $i$ th rows of  $M$  and  $S$  appropriately.

Increased sensor noise failure:

$$z(k) = Hx(k) + Ju(k) + v(k) + \xi(k)\sigma_{k+1,0} \quad (51)$$

The analysis of these failure modes is presently being performed (refs. 48 and 50), and it is anticipated that SGLR algorithms will also be developed.

In addition to these failure modes, one can develop additional models along these lines for particular applications. In particular, we have developed several additional models similar to those described by equations (40) to (44) for our work on the detection and classification of arrhythmias in electrocardiograms. The results reported in reference 21 are rather striking; as in all the tests performed we observed no false alarms, detected all rhythm changes immediately (with no incorrect estimates of  $\theta$ ), and classified all rhythm changes correctly. These tests used the full GLR approach and have provided useful insight into the characteristics of the method. For example, the use of maximum likelihood estimates of  $v$  and  $\theta$  precludes the use of *a priori* statistics on these variables. In the ECG problem, one is quite interested in accurate estimates of  $v$ , and one also can come up with reasonable *a priori* statistics on  $v$  based on physical arguments. Thus, it may pay to incorporate such *a priori* statistics into the GLR system, and this can be done rather easily by proper initialization of the matched filters estimating  $v$ . On the other hand, for the ECG problem, one does not want to look for abrupt changes at one point in the record more than at another, and thus it does not make sense to include *a priori* statistics on  $\theta$ . In fact, one can argue that inclusion of *a priori* failure information tends to discount the observed data to avoid false alarms (unless failures are extremely likely), and one should probably avoid the inclusion of such information unless one is especially worried about false alarms. However, if one wishes to use such data, one can use the interpretation of the likelihood ratios as ratios of conditional probabilities of failure times to determine the appropriate modification of GLR (ref. 45).

Finally, note that the GLR system provides extremely useful information for system compensation subsequent to the detection of a failure. For example, one can use the GLR-produced estimates of  $v$  and  $\theta$  to determine an optimal update procedure for the filter estimate and covariance (ref. 45). Once this update has been performed, the GLR system can be used to detect further failures, thus allowing the detection of multiple events. (See refs. 45 and 48 for further discussions of the use of GLR-produced information in the design of failure compensation systems.)

## VIII. CONCLUSIONS

We have discussed a number of the issues involved in the design of failure detection systems. We have also reviewed a variety of existing failure detection methods and have discussed their characteristics and design trade-offs. The failure detection problem is an extremely complex one, and the choice of an appropriate design depends heavily on the particular application. Issues such as available computational facilities and level of hardware redundancy enter in a crucial way in the design decision. For example, as we have mentioned, the use of a sophisticated failure detection-compensation



system may allow one to reduce the level of hardware redundancy without much loss in overall system reliability.

The development of failure detection methods is still a relatively new subject. At this time, most of the work has been at a theoretical level with only a few real applications of techniques (refs. 6, 14-17, 21, and 23). Much work remains to be done in the development of implementable systems complete with a variety of design tradeoffs. Work is needed in the development of efficient techniques for failure compensation and system reorganization. In addition, there is a great need for the analysis of the robustness of various failure detection systems in the presence of variations in system parameters and in the presence of modeling errors and system nonlinearities. For example, it is conjectured that SGLR is less sensitive to parameter errors than the more complex full GLR. However, presently there are no analytical results or simulations to support this conjecture. These and other issues, such as the incorporation of fault-tolerant computer concepts into an overall reliable design methodology (see ref. 51) await investigation in the future.

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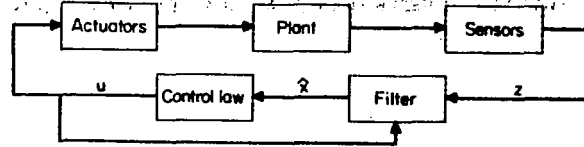


Figure 1.- No-failure system configuration.

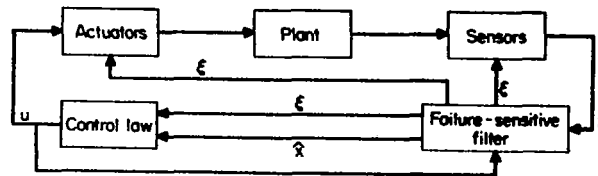


Figure 2.- Failure detection system involving failure-sensitive primary filter ( $\xi$  denotes information concerning detected failures).

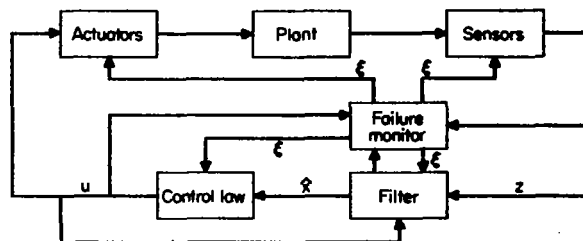


Figure 3.- Failure detection system involving a monitoring system for no-failure configuration.





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